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Measuring Natural Rate of Interest in Uzbekistan

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Abstract

The natural rate of interest, often interpreted as the equilibrium real interest rate, serves as a critical benchmark for evaluating the stance of monetary policy. This paper investigates the natural rate of interest in Uzbekistan using three econometric approaches: the HLW-type model[1], a modified HLW-type model, and the Central Bank of Uzbekistan's Quarterly Projection Model (QPM), semi-structural framework. The semi-structural HLW-type and modified HLW-type models estimate the real interest rate using core inflation, while the QPM relies on headline inflation. The results indicate that the natural rate was relatively stable across the models. The semi-structural HLW-type and modified HLW-type models produced average natural rate estimates of 4.5 percent and 4.1 percent, respectively, while the QPM estimated a slightly lower average rate of 3.4 percent. On average, the natural rate across all models was approximately 4.0 percent. In contrast, the real interest rate exhibited significant variability, reflecting periods of accommodative monetary policy before the adoption of the inflation targeting regime and restrictive policies during the IT regime's active implementation. These findings emphasize the critical role of accurately estimating the natural rate to guide monetary policy and ensure macroeconomic stability effectively.

Keywords: Natural level of Real Interest Rate, Core Inflation, Kalman Filter, Bayesian Estimation.

JEL: E42, E43, E520, E590

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[1] Holston-Laubach-Williams (2016)

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1 Introduction

The natural rate of interest is defined as the real interest rate at which monetary policy neither stimulates nor restrains economic activity, maintaining a balance between inflation and output. Estimating this rate has become an important task for Uzbekistan as the country undergoes major economic changes.

Since 2019, the Central Bank of Uzbekistan (CBU) has shifted its monetary policy regime to inflation targeting (IT), focusing on achieving price stability. Preparations for this shift began in 2017 with the liberalization of the foreign exchange market—an essential step, as inflation-targeting frameworks generally require a floating exchange rate regime. In 2019, with the introduction of a new law on the central bank, the CBU's primary goal was redefined from ensuring the stability of the national currency to ensuring price stability, with an inflation target set at 5 percent.

The active phase of the inflation-targeting regime began in 2020, with the CBU adopting the key rate as its main policy instrument, as is typical in such frameworks. In this context, the natural rate of interest serves as a crucial guide for determining the appropriate policy stance to achieve and sustain the inflation target while supporting balanced economic growth.

Since 2020, in order to increase the flexibility of monetary policy tools, the Central Bank introduced the key interest rate and interest rate corridor, and moved to a system where the key rate is reviewed eight times a year.

In improving the operational mechanism of the Central Bank, deposit auctions, overnight deposits, Central Bank bonds, liquidity-providing swaps, repo auctions, and overnight operations were introduced. Additionally, new indicator system reflecting changes in the money market as a benchmark rate, called the 'UZONIA'[2] index, was developed.

The inflation rate was reduced from 18 percent in 2018 to 11.1 percent in 2020, and by the end of 2021, annual headline inflation reached 10 percent, meeting the set interim target. Due to external risks that emerged in 2022, annual inflation rose to 12.3 percent, but it decreased to 8.8 percent in 2023 and it is expected to reach the 5 percent target by the end of 2026[3].

The CBU's primary tool for policy guidance is the Quarterly Projection Model (QPM), which provides forecasts and guidance on the policy rate for monetary policy decisions. Within the QPM, a modified Taylor rule is employed to guide policy rate decisions. An interest rate above the natural rate indicates a tight monetary policy stance, which typically leads to slower growth, and declining inflation. Conversely, an interest rate below the natural rate indicates an accommodative monetary policy stance, fostering faster growth, and increasing inflation. As of January 2025, CBU's policy rate stands at 13.5 percent.

Monetary policy transmission mechanism in Uzbekistan has historically faced challenges due to structural constraints. A significant factor has been the dominance of state-owned banks, which account for 67 percent of total banking sector assets. These banks have not fully operated on market principles, limiting the transmission of changes in the policy rate to retail lending and deposit rates. Additionally, the high degree of dollarization — 25.5 percent of deposits and 41.1 percent of credit as of January 2025 — further weakens monetary policy effectiveness by reducing the influence of the domestic currency. Other factors, such as a shallow financial market and preferential government lending programs, have also diluted the impact of monetary policy. However, ongoing reforms efforts to reduce dollarization, strengthens transmission channels, improves the effectiveness of monetary policy in achieving its objectives.

[2]UZONIA (Uzbek Overnight Index Average) – is an interest indicator, calculated as the weighted average interest rate for overnight deposit operations between commercial banks in the national currency in the unsecured interbank money market.

[3]Monetary Policy Guidelines for 2025 and 2026-2027, CBU

Building on methods used in studies of similar economies, this research estimates natural rate of interest for Uzbekistan by combining different models. This includes a modified Laubach-Williams model, the Taylor rule for small open economies, and the CBU's own QPM. Each model provides valuable perspectives: the modified Laubach-Williams model reflects the structural change in Uzbekistan's economy; the Taylor rule is a straightforward approach to adjusting interest rates based on inflation and output gaps; and the QPM captures specific features of Uzbekistan's monetary policy transmission. Together, these approaches offer a well-rounded estimate of the natural rate suited to Uzbekistan's economic environment.

By estimating Uzbekistan's natural rate of interest, this study aims to assist the CBU in setting effective monetary policy in its ongoing transition. This research can help strengthen Uzbekistan's progress towards price stability and support sustainable economic growth.

2 Literature review

2.1 Fundamentals of the Natural Rate of Interest

The natural rate of interest, often referred to as the equilibrium real interest rate, is a foundational concept in modern macroeconomic theory and monetary policy. It serves as a critical benchmark for evaluating the stance of monetary policy. This concept was first introduced by Knut Wicksell (1898) in his seminal work, Interest and Prices, where he described it as "there is a certain rate of interest on loans which is neutral in respect to commodity prices, tending neither to raise nor to lower them."

The concept gained prominence in macroeconomics during the 20th century, particularly in the context of monetary policy discussions. It was reintroduced into modern economic discourse by economists such as John B. Taylor and has since become central to the analysis of policy frameworks, including the formulation of the Taylor Rule.

In recent years, the natural rate of interest has garnered renewed attention, particularly in the aftermath of the COVID-19 pandemic. The sharp rise in inflation and the corresponding global shift toward higher interest rates have brought the natural rate of interest to the forefront of policy debates. Policymakers and researchers are increasingly focused on assessing whether these elevated rates are sufficiently restrictive to curb inflation without excessively dampening economic growth.

The natural rate of interest is rooted in the equilibrium framework of economic theory, often derived from models such as the New Keynesian framework and growth theories. At its core, r^* is the rate that balances savings and investment in the economy over the long term, ensuring macroeconomic stability. Mathematically, r^* can be expressed in a simplified form as:

$$r^* = \rho + g$$

where ρ[4] represents time preferences for current versus future consumption, and g denotes the long-term growth rate of the economy.

This equation highlights how r^* is influenced by factors such as productivity growth, demographic trends, and preferences for saving versus consumption. However, in practice, r^* is time-varying, reflecting changing economic conditions and external influences.

Key Drivers in Advanced Economies

In advanced economies, r^* has experienced a long-term decline due to several structural and cyclical factors in recent decades. Demographic changes, particularly aging populations in advanced economies such as Japan, Europe, and North America, have been shown to influence the natural rate of interest by increasing savings rates while simultaneously shrinking the labor force. This demographic transition reduces the growth rate of the potential output and exerts downward pressure on the equilibrium interest rate r^*. The reduction in the working-age population constrains economic output, while higher savings among aging households amplify the decline in r^* (Carvalho et al., 2016; Eggertsson et al., 2019; Rachel and Summers, 2019).

The decline in productivity growth, particularly pronounced since the 2008 financial crisis, has significantly curtailed potential output, a key determinant of the natural rate of interestr^*. Lower productivity growth reduces the marginal return on capital, discouraging investment, and exerting downward pressure on equilibrium rates (Summers, 2014). This trend has been observed across advanced economies, as documented by Fernald (2015), who highlights slowing technological progress as a critical factor. Furthermore, Holston, Laubach, and Williams (2017) emphasize that persistent declines in productivity are a primary driver of the long-term decline in r^*.

[4]Discount factor is assumed to be in the form of $e^{-\rho t}$

The global savings glut, fueled by rising precautionary savings in aging economies and persistent current account surpluses in countries such as China, has placed sustained downward pressure on interest rates. This phenomenon, highlighted by Bernanke (2005), reflects an excess of global savings relative to investment opportunities. Advanced economies, often viewed as safe havens, attract substantial capital inflows during periods of global uncertainty, further suppressing the natural rate of interest r^* (Caballero, Farhi, and Gourinchas, 2008; Rachel and Smith, 2015). This dynamic has been particularly pronounced in the aftermath of the 2008 financial crisis, with heightened demand for safe assets exacerbating the decline in equilibrium rates.

Periods of increased economic uncertainty and elevated financial market risks lead to an increase in precautionary savings and a simultaneous reduction in investment. These dynamics contribute to persistently low levels of the natural rate of interest r^*. As Bloom (2009) highlights, uncertainty shocks can significantly delay investment decisions due to the option value of waiting, thereby reducing capital accumulation. Similarly, Caballero and Farhi (2017) emphasize that the demand for safe assets increases during uncertain times, leading to a higher global savings rate and downward pressure on equilibrium interest rates. These effects are further reinforced during crises, as observed by Gourinchas and Rey (2016), when risk-averse behavior dominates financial markets and economic agents prioritize liquidity and safety over productive investment.

Prolonged periods of accommodative monetary policy, implemented in response to economic shocks, have significantly shaped market expectations and contributed to the decline in estimates of the natural rate of interest r^*. Holston, Laubach, and Williams (2017) argue that persistent low policy rates in advanced economies have anchored expectations of lower future equilibrium rates. Similarly, Rachel and Smith (2015) highlight that unconventional monetary policies, such as quantitative easing, have amplified this trend by increasing the demand for safe assets, further suppressing r^*

Key Drivers in Emerging Economies

Emerging economies, such as Uzbekistan, show different drivers of the natural rate of interest r^* due to their unique growth trajectories, developmental challenges, and vulnerabilities to external factors. The interaction of these elements shapes the equilibrium interest rate in ways that differ from those of advanced economies. Emerging economies tend to exhibit higher growth rates than advanced economies, driven by structural transformations, technological catch-up, and capital accumulation. According to Barro (1991), these factors often lead to higher productivity growth, which, in turn, raises the natural rate of interest. As countries modernize their economies and improve infrastructure, the demand for capital increases, pushing r^* higher. In particular, emerging economies benefit from the "catch-up effect" where productivity growth outpaces that of advanced economies (Abrams, 2005).

The dependence on foreign direct investment (FDI) and external financing in emerging economies makes r^* particularly sensitive to global financial conditions. A study by Kose et al. (2009) underscores the significant role of capital flows in determining interest rates in developing economies, especially during periods of heightened global financial volatility. Additionally, fluctuations in risk premiums and exchange rate stability, often influenced by global events, play a crucial role in shaping the equilibrium interest rate in these economies (Calvo et al., 1993).

Younger populations in emerging markets tend to generate higher investment demand, driven by increasing labor force participation and urbanization. This demographic dividend can push the natural rate of interest higher as the demand for capital rises to support rapid urbanization and infrastructure development. According to Bloom et al. (2011), a growing working-age population accelerates investment in physical and human capital, creating upward pressure on r^*.

Structural reforms aimed at improving fiscal sustainability, governance, and monetary policy independence can affect the potential growth trajectory and, by extension, r^*. As highlighted by Alesina et al. (2005), countries undergoing substantial institutional reforms often experience shifts in their natural rate of interest. Improved fiscal policy frameworks and greater monetary policy credibility can lower risk premiums and enhance capital flows, contributing to a higher r^*. In contrast, policy uncertainty or political instability can suppress investment and reduce the natural rate of interest.

Emerging economies are particularly vulnerable to external shocks, such as commodity price volatility, fluctuations in remittance flows, and geopolitical risks. These factors can create significant variability in the natural rate of interest. For example, commodity price fluctuations, as noted by Blanchard et al. (2017), can have direct effects on capital accumulation and productivity, influencing the equilibrium interest rate in resource-dependent economies. Additionally, geopolitical risks can lead to capital outflows and exchange rate instability, both of which can depress r^* in the short run.

Common Global and Regional Influences

Both advanced and emerging economies are increasingly influenced by global and regional factors that shape the dynamics of r^*. These common drivers reflect the interconnectedness of the global economy and its impact on national interest rates. The integration of economies into global trade and financial networks has resulted in synchronized interest rate dynamics. Global savings and investment trends now play an important role in shaping interest rates in regions. According to Obstfeld and Rogoff (1996), globalization has led to a convergence of interest rates across countries, making both advanced and emerging markets sensitive to global economic conditions. The cross-border flow of capital influences national interest rates, amplifying the effect of global savings glut on both advanced and developing economies.

The transition to green energy and the broader push toward sustainability are reshaping investment needs and resource allocation, with significant long-term implications for r^*. According to the OECD (2020), the global shift towards environmentally sustainable practices is expected to drive increased demand for green investments and infrastructure, which could influence the natural rate of interest by shifting investment patterns and the overall supply and demand for capital. This transition will affect both advanced and emerging economies differently, depending on their capacity to invest in clean technologies.

While advanced economies lead in innovation, emerging markets benefit from technology diffusion, which can have differing effects on productivity growth across regions. As noted by Comin and Hobijn (2009), technological spillovers from developed to developing countries can increase productivity growth, thereby raising the natural rate of interest in emerging economies. The pace and extent of technological adoption in emerging markets is critical in determining whether these economies will experience a rise or fall in r^*.

Global economic policy coordination, particularly in monetary and fiscal policies, has an indirect but significant impact on r^*, especially in economies that are highly open and sensitive to global interest rate movements. The coordination of policy responses, as seen during the global financial crisis, can create a global liquidity effect that impacts interest rates in both developed and emerging economies (Taylor, 2009). As global central banks implement similar policy measures, such as low-interest rates or quantitative easing, the global rate of interest influences national rates, contributing to the persistence of low r^* globally.

2.2 Empirical Studies on the Natural Rate of Interest

One of the most widely used approaches to estimate the natural rate of interest is the Laubach-Williams (LW) model, introduced in 2001. This small semistructural model was initially developed for the US economy and is designed to estimate the unobservable natural rate of interest by jointly modeling the relationships between potential output, trend growth, and inflation dynamics. The LW model assumes a state-space representation, where the natural rate and potential output are treated as latent variables and are estimated using the Kalman filter. The findings of Laubach and Williams showed that the natural rate of interest in the US declined significantly over the decades leading up to their analysis. This decline was primarily attributed to slower trend productivity growth and demographic factors that reduced the equilibrium level of savings and investment. These results emphasized the importance of incorporating long-term structural changes into monetary policy decisions. Over time, the model has been extended and adapted for use in other economies, including emerging markets and small open economies, making it a versatile tool for policymakers.

Global trends in the natural rate have also been explored extensively. Holston et al. (2017) provide a comparative analysis across major economies, illustrating the influence of demographic shifts, productivity trends, and global financial integration on the long-term decline in natural rates. Their methodology, grounded in the Kalman filter, offers insights into the persistent comovement of natural rates across economies, underscoring the significance of global factors.

Buncic (2024) highlights the econometric challenges in estimating the natural rate using widely adopted structural models such as the one developed by Holston, Laubach, and Williams (2017). A key issue identified is the spurious downward trend in estimates due to the misspecification of signal-to-noise ratios, leading to biased results. Addressing these issues is crucial for improving the reliability of natural rate estimates, especially for policy applications in advanced economies.

For developing countries, the estimation of the natural rate involves additional complexities, as highlighted in studies like Teodoru and Toktonalieva (2020) on the Kyrgyz Republic. These challenges stem from higher public debt, elevated risk premiums, and structural inefficiencies that differentiate these economies from their advanced counterparts. Such conditions necessitate the adoption of tailored frameworks, such as semi-structural models or quarterly projection models, to capture the dynamics of the natural rate effectively.

Emerging economies with small open structures, like the Czech Republic, require models that account for external influences. Hlédik and Vlček (2018) emphasize the role of equilibrium real exchange rate appreciation alongside potential output growth in determining the natural rate for such economies. Their findings reveal that changes in global capital flows and trade dynamics significantly affect the natural rate in small open economies, necessitating a comprehensive approach that integrates these external factors.

Finally, in contexts like Denmark, where fixed exchange rate regimes constrain monetary policy, the natural rate plays a pivotal role in evaluating imported monetary policy. Pedersen (2015) explores the implications of a negative natural rate for Denmark, highlighting the risks of secular stagnation and the need for structural reforms to enhance resilience against cyclical shocks.

In summary, understanding the natural rate of interest is essential for designing effective monetary policies, especially in developing economies. As these studies illustrate, the methodologies for estimating the natural rate must adapt to the structural characteristics and external vulnerabilities of each economy, ensuring that policymakers have reliable benchmarks to guide their decisions.

2.3 Implications for Uzbekistan

In the context of Uzbekistan, the natural rate of interest is influenced by a combination of domestic and global factors. In light of these considerations, this study employs the widely-used HLW-type model to examine how potential growth and inflation influence the natural rate of interest. Additionally, a modified HLW model incorporating a Taylor-type rule is utilized to capture monetary policy responses within the framework and incorporates real effective exchange rate movements. While HLW-type models are particularly suited for advanced economies with extensive historical time series, their application to small open economies like Uzbekistan presents challenges due to the limited availability of long-term data.

Given the constraints of limited long-term data, incorporating additional indicators such as fiscal and monetary policy variables, remittances, wage dynamics, exchange rate movements, foreign direct investment, and other relevant economic factors can enhance the precision of estimates.

These variables provide critical insights into the underlying economic structure and external influences, allowing for a more comprehensive and accurate assessment of the natural rate of interest.

To address these limitations, the QPM of CBU is also employed. The QPM, with its rich structural design, offers a robust framework that accommodates shorter data horizons while providing insights into the broader macroeconomic environment. This makes it a valuable complementary tool for estimating in the context of Uzbekistan's unique economic dynamics.

Another important consideration is the shift in Uzbekistan's monetary policy regime in 2020, when the CBU adopted an IT framework. While monetary policy regimes can influence short-term interest rates and economic conditions, the natural rate is primarily determined by slow-moving structural factors that are less sensitive to changes in policy frameworks. This raises the question of whether a monetary policy regime switch, such as the transition to inflation targeting (IT), has a significant impact on the estimation of r^*. In the context of a regime switch, such as the adoption of inflation targeting, the primary impact is on the central bank's policy rate and its transmission mechanisms. However, if the structural determinants of r^*— such as potential output growth, inflation expectations, and savings rates—remain relatively stable, the natural rate itself may not exhibit significant changes. This suggests that the estimation of r^* may not require explicit adjustments for regime switches, provided that the underlying economic fundamentals remain unchanged.

In the case of Uzbekistan, the transition to an inflation targeting regime in 2020 marked a significant shift in the Central Bank of Uzbekistan's (CBU) monetary policy framework. However, if the structural determinants of r^* —such as potential output growth, inflation expectations, and savings behavior—have remained relatively stable, the regime switch may not have a substantial impact on the estimation of the natural rate. This is consistent with the findings of this study, which show that the natural rate of interest in Uzbekistan has remained relatively stable across different models, with average estimates ranging from 3.4 to 4.5 percent.

Moreover, the real interest rate, which reflects the CBU's policy stance, has exhibited significant variability, particularly during periods of external shocks and structural reforms, such as the liberalization of the foreign exchange market in 2017. This divergence between the natural rate and the real rate further supports the argument that r^* is influenced more by long-term structural factors than by short-term policy changes.

One way to check the robustness of these findings would be to estimate the natural rate of interest separately for the periods before and after the regime switch in 2020 and conduct a Chow test (or other tests such as the Bai-Perron tests) to detect structural breaks. However, the short data sample poses significant challenges for such an approach. HLW-type models, which rely on long time series to capture the slow-moving dynamics of potential output, inflation expectations, and other structural factors, may not yield reliable results when applied to small subsamples.

Given these limitations, it is more appropriate to rely on the whole sample estimate, even though the sample itself is not particularly long. This approach is also justified by the fact that key structural determinants of r^*, such as potential output growth, inflation expectations, and savings behavior, have remained relatively stable during the observed period. The absence of significant changes in these underlying factors suggests that the natural rate of interest is unlikely to have been substantially affected by the regime change.

While the whole sample estimate may not fully capture the nuances of the regime switch, it provides a more stable and consistent framework for estimating r^* in the context of Uzbekistan's evolving economic environment. This approach balances the need for robustness with the practical constraints imposed by data availability and the stability of the structural drivers of r^*.

3 Model identification

3.1 Semi-structural Model 1 (HLW-type)

We adapt the model to introduce a connection between the permanent components of output and the interest rate. The modified model retains the core structure of Laubach Williams(2016). but estimation approach of semi-structural use Bayesian method.

Model Equations and Definitions

The output (y_t) is divided into gap and trend:

$$y_t = \tilde{y}_t + \bar{y}_t \tag{1}$$

where y_t is constructed as 400 times the log of real GDP.

The output gap $(\tilde{y}_t)$, which represents the difference between actual and potential output, is influenced by its lags and the real interest rate rate gap $(\tilde{r}_t)$, defined as the deviation of the interest rate from its natural level. Positive values indicate an overheated economy, while negative values signal under performance.

$$\tilde{y}_t = a_y(L)\tilde{y}_{t-1} + a_r(L)\tilde{r}_{t-1} + \eta_{\tilde{y},t} \tag{2}$$

where $a_y(L) = a_1 L + a_2 L^2$ and $a_r(L) = a_3$ are lag polynomials, and $\eta_{\tilde{y},t}$ is a Gaussian white noise error term. The interest rate gap influences the output gap with a one-period lag.

Potential output, y_t^*, follows a random walk with drift, representing its long-run stochastic trend:

$$y_t^* = y_{t-1}^* + g_t + \eta_{y^*,t} \tag{3}$$

$$g_t = g_{t-1} + \varepsilon_{g,t} \tag{4}$$

where g_t is the stochastic growth rate of potential output, and $\varepsilon_{g,t}$ and $\eta_{y^*,t}$ are random disturbances.

Inflation in these models is often described by a Phillips curve that relates inflation to expected inflation and the output gap where inflation rises when output exceeds potential levels. The Phillips curve can be expressed as:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\mathbb{E}_t \pi_t + b_y \tilde{y}_{t-1} + \epsilon_{\pi,t} \tag{5}$$

where $b_\pi = b_1$, $b_y = b_2$ are parameters determines the share of forward looking versus backward looking agents in the core-goods markets and the impact of real marginal costs on core inflation respectively; π_t represents inflation, $\mathbb{E}_t\pi_t$ (defined as $(\pi_{t-2} + \pi_{t-3} + \pi_{t-4})/3)$) is expected inflation and $\epsilon_{\pi,t}$ is a random error term. The interest rate affects inflation indirectly, through the output gap.

Natural rate of interest r_t^* which is the real interest rate consistent with an economy operating at its full potential, with inflation at target, is represented as a random walk with a stochastic trend, ζ_t, and is influenced by the growth rate of the real GDP trend. It reflects long-term equilibrium levels and adjusts for cyclical shocks. The relationship between potential growth and the natural rate of interest in emerging economies is shaped by several key factors, which may not result in a one-to-one relationship. According to Laubach and Williams (2003), the natural rate of interest is influenced not only by growth but also by capital accumulation, savings preferences, and external shocks. For instance, while high growth rates may signal higher investment demand, structural weaknesses such as limited financial development, political instability, and reliance on volatile capital flows can offset the positive effect on r^*. Additionally, emerging economies often experience substantial external influences—such as commodity price fluctuations and shifts in global capital flows—that may dampen or amplify the impact of potential growth on the natural rate of interest. Therefore, we have modified the relationship with potential and natural rate:

$$r_t^* = b * g_t + \zeta_t \tag{6}$$

$$\zeta_t = \zeta_{t-1} + \varepsilon_{\zeta,t} \tag{7}$$

where b is a constant which shows how much from potential growth affects to r^*; $\varepsilon_{\zeta,t}$ are *i.i.d.* random error terms.

This specification implies that the interest rate is non-stationary, which is consistent with much of the literature that finds natural rates exhibit unit root behavior. Real interest rate r_t is calculated using overnight interbank rate i_t:

$$r_t = \ i_t - \pi_t^e \tag{8}$$

where π_t^e is inflation expectation defined as average of last 4 quarters inflation:

$$\pi_t^e = 1/4 * (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}) \tag{9}$$

3.2 Semi-structural Model 2 (modified HLW-type model)

One limitation of the HLW model is that it is primarily tailored for advanced economies, where the data aligns well with the assumptions of the New Keynesian framework over extended time periods.[5] Moreover, in HLW model the role of the external sector is minimal. In the context of a small open economy, however, the exchange rate plays a crucial role in influencing inflation, trade balances, and overall economic dynamics. The LW model does not explicitly account for exchange rate fluctuations, making it less suitable for capturing the unique characteristics and challenges of small open economies. We modify Laubach Williams (2017) by accounting for a real effective exchange rate and closing the semi-structural model using the Taylor type rule for monetary policy.

Dynamic Taylor rules establish a relationship between the policy interest rate and economic indicators such as inflation, the output gap, and the real exchange rate gap. The general form of the Taylor rule is expressed as:

$$i_t = i_t^* + \beta(\pi_t - \pi_t^*) + \theta\tilde{y}_t + \phi r\tilde{eer}_t + \epsilon_t, \tag{10}$$

where i_t is policy interest rate, i_t^* is natural nominal interest rate, $(\pi_t - \pi_t^*)$ is deviation of inflation from its target, $\tilde{y}_t$ is output gap, $r\tilde{eer}_t$ is real effective exchange rate gap, ϵ_t is stochastic disturbance term.

The natural nominal rate follows is defined as following:

$$i_t^* = r_t^* + \pi_t^*, \tag{11}$$

where r_t^* is natural rate of interest, and π_t^* is inflation target.

An augmented version of the Taylor rule includes monetary policy persistence:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left(i_t^* + \beta(\pi_t - \pi_t^*) + \theta\tilde{y}_t + \phi r\tilde{eer}_t\right) + \epsilon_t, \tag{12}$$

where ρ_i represents the degree of persistence in monetary policy decisions.

We augment the equation for the output gap with a real effective exchange rate gap as follows:

$$\tilde{y}_t = a_y(L)\tilde{y}_{t-1} + a_e(L)\tilde{r}_{t-1} + a_{reer} r\tilde{eer}_{t-1} + \eta_{\tilde{y},t} \tag{13}$$

[5] IMF, 2023 World Economic Outlook: A Rock y Recovery. CH2: The natural rate of interest: drivers and implications for policy

The trend and gap components are defined as a random walk and AR(2) process:

$$r\bar{e}er_t = r\bar{e}er_t + \varepsilon_{reer,t} \tag{14}$$

$$r\tilde{e}er_t = c_1 * r\tilde{e}er_{t-1} + c_2 * r\tilde{e}er_{t-2} + \epsilon_{reer,t} \tag{15}$$

Other equations are taken as in the Structural model 1.

3.3 Semi-structural model 3 (QPM model of CBU)

Following the adoption of the IT regime, CBU developed a QPM to analyze the current state of the economy and forecast its future trajectory. The QPM is a forward-looking, semi-structural New-Keynesian framework tailored to small open economies. It belongs to a family of QPM-type models widely utilized by central banks and international organizations globally. These models serve as simplified representations of structural dynamic stochastic general equilibrium (DSGE) models, enhanced with data-driven extensions. They combine the narrative strengths of DSGE models with the adaptability of reduced-form models, making them highly suitable for practical policy-making. Although QPMs lack the detailed microeconomic foundations of DSGE models, they are more flexible, user-friendly, and often provide a better fit to empirical data. Known as gap models, they represent real variables as deviations from their long-term sustainable levels. The CBU's QPM follows a standard structure while incorporating features specific to Uzbekistan's economic context.

Aggregate Demand - IS-curve

The aggregate demand gap is influenced by its previous value, the real monetary conditions index, the foreign output gap, and various other macroeconomic factors, as represented by the following equation:

$$\begin{aligned} \hat{y}_t = a_1\hat{y}_{t-1} - a_3 rmci_{t-1} + a_4\hat{y}_t^f + a_5(\hat{r}em_{t-1} + \hat{z}_{t-1}) + a_6 fiscimp_t + \\ + a_7\frac{\hat{cr}_{new,t}}{4y_t^{nom}} + a_8\frac{\hat{fdi}_t}{4y_t^{nom}} + \varepsilon_t^{\hat{y}} \end{aligned} \tag{16}$$

where $\hat{y}_t$ - output gap, a_1 - persistence parameter, capturing the lagged effect of the output gap; $rmci_{t-1}$ - real monetary conditions index, reflecting the combined effect of the real interest rate gap and the real exchange rate gap on aggregate demand; $\hat{y}_t^f$ - foreign output gap, weighted by trade partner significance; $\hat{r}em_{t-1}$ - real remittance gap, representing the deviation of remittances from their long-term trend, adjusted for the exchange rate; $\hat{z}_{t-1}$ - real exchange rate gap, capturing the deviation of the real exchange rate from its long-run trend; $fiscimp_t$ - fiscal impulse, indicating the impact of fiscal policy adjustments on aggregate demand; $\frac{\hat{cr}_{new,t}}{4y_t^{nom}}$ - new credit-to-GDP gap, reflecting the deviation of newly issued credit from its long-run share of nominal GDP; $\frac{\hat{fdi}_t}{4y_t^{nom}}$ - FDI-to-GDP gap, measuring the deviation of foreign direct investment from its long-run share of nominal GDP; $\varepsilon_t^{\hat{y}}$ - structural demand shock.

Real Monetary Conditions Index

$$rmci_t = a_{rmci}(-\hat{z}_t) + (1 - a_{rmci})\hat{r}_t \tag{17}$$

where $rmci_t$ - real monetary conditions index; a_{rmci} - weight parameter for the exchange rate gap in the monetary conditions index; $-\hat{z}_t$ - negative of the real exchange rate gap, which indicates appreciation or depreciation relative to the long-run trend; $\hat{r}_t$ - real interest rate gap, capturing the deviation of the real interest rate from its equilibrium.

Potential Output Growth

$$\begin{aligned} \Delta y_t = c_{\Delta y}\Delta y_{t-1} + (1 - c_{\Delta y})\Delta y_t^{ss} + c_{2\Delta y}\left(\frac{fdi_t}{4y_t^{nom}} - \frac{fdi^{ss}}{4y_t^{nom}}\right) + \\ + c_{3\Delta y}(\Delta gold_t - \Delta gold^{ss}) + \varepsilon_t^{\Delta y} \end{aligned} \tag{18}$$

where Δy_t - growth rate of potential output; $c_{\Delta y}$ - persistence parameter for potential output growth; Δy_t^{ss} - steady-state growth rate of potential output; $\frac{fdi_t}{4y_t^{nom}}$ - FDI-to-GDP ratio, with fdi^{ss} as its steady-state; $\Delta gold_t$: growth rate of real gold prices, adjusted by the steady-state growth $\Delta gold^{ss}$; $\varepsilon_t^{\Delta y}$ - shock to potential output growth.

Phillips Curve - Core Inflation

$$\pi_t^{core} = b_{core1}\mathbb{E}_t\pi_{t+1}^{core} + (1 - b_{core1} - b_{core2})(b_{adapt}\pi_{t-1}^{core} + (1 - b_{adapt})(\pi_{t-1} + \pi_t^{rpcore}))+ \\ +b_{core2}\pi_t^{imp} + b_{core3}rmci_t^{core} + \varepsilon_t^{\pi^{core}} \tag{19}$$

where π_t^{core} - core inflation, capturing underlying inflation trends in the economy; b_{core1} - forward-looking parameter for inflation expectations; $\mathbb{E}_t\pi_{t+1}^{core}$ - expected core inflation in the next period; b_{core2} - imported inflation weight; π_t^{imp} - imported inflation; b_{core3} - coefficient for real marginal cost, $rmci_t^{core}$, representing inflationary pressures from the cost side; $\varepsilon_t^{\pi^{core}}$ - shock to core inflation.

Policy Interest Rate

The central bank's policy interest rate is based on a Taylor-type rule, formulated as:

$$i_t^p = c_{i^p}i_{t-1}^p + (1 - c_{i^p})\left(r_t + c_{i^p2}dev_t^{cpi} + c_{i^p3}\hat{y}_t\right) + \varepsilon_t^{ip} \tag{20}$$

where i_t^p - nominal policy interest rate; i_{t-1}^p - lagged policy interest rate, capturing the gradual adjustment of monetary policy; r_t - natural rate of interest; dev_t^{cpi} - deviation of year-on-year inflation from the target; $\hat{y}_t$ - output gap, a measure of economic activity relative to potential output; c_{i^p} - interest rate smoothing coefficient; c_{i^p2}, c_{i^p3} - weights for inflation deviation and output gap, respectively; ε_t^{ip} - monetary policy shock.

The deviation of year-on-year inflation from the target is defined as:

$$dev_t^{cpi} = \pi_{t+2}^{yoy} - \pi_{t+2}^{tar}, \tag{21}$$

where π_{t+2}^{yoy} - expected year-on-year inflation two periods ahead; π_{t+2}^{tar} - inflation target two periods ahead.

The natural rate of interest, $\bar{r}_t$, is determined by:

$$\bar{r}_t = c_r\bar{r}_{t-1} + (1 - c_r)\bar{r}_t^{uip} \tag{22}$$

where $\bar{r}_t$ - natural rate of interest; $\bar{r}_{t-1}$ - lagged natural rate; $\bar{r}_t^{uip}$ - UIP-based real interest rate trend.

Real Interest Rate The real interest rate is derived using the Fisher equation:

$$r_t = i_t - \mathbb{E}_t\pi_{t+2}^4 \tag{23}$$

where π_{t+2}^4 is expected year-on-year inflation over two quarters ahead.

The UIP-based real interest rate trend is expressed as:

$$\bar{r}_t^{uip} = \bar{r}_t^f + \mathbb{E}_t\Delta\bar{z}_t^{us} + prem_t \tag{24}$$

where $\bar{r}_t^f$ - foreign real interest rate trend; $\mathbb{E}_t\Delta\bar{z}_t^{us}$ - expected growth of the real exchange rate trend against the USD.

The UIP risk premium evolves according to an autoregressive process:

$$prem_t = c_{prem1}prem_{t-1} + (1 - c_{prem1})prem^{ss} + \varepsilon_t^{prem} \tag{25}$$

where $prem^{ss}$ - steady-state UIP risk premium; ε_t^{prem} - shock to the UIP risk premium.

4. Data and Estimation Methodology

4.1 Data

Quarterly data for Uzbekistan, starting from 2006Q2 to 2024Q3, is utilized for core inflation (seasonally adjusted and annualized quarter-on-quarter inflation excluding prices of fruits, vegetables, and regulated goods) and GDP (log-transformed and scaled by 400 for annualizing). These data are sourced from the Statistics Agency database and are employed in both the HLW-type and modified HLW-type semi-structural models (Figure 1). Core inflation is used in the LW model as it reflects the underlying inflation trend, excluding components with high volatility, making it more suitable for capturing the persistent dynamics of inflation.

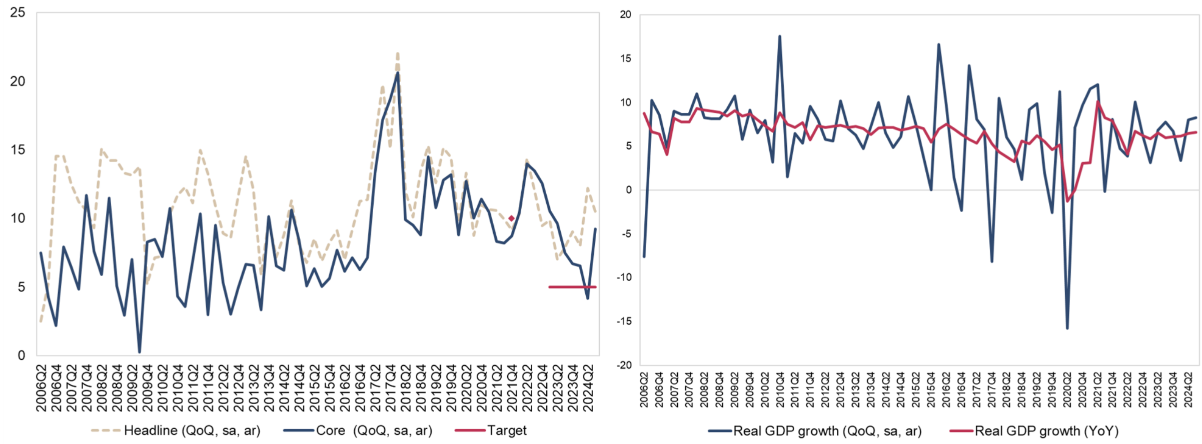

Figure 1: Inflation and GDP growth

During the period 2006-2016 core inflation and real GDP growth were relatively stable around 6-7 percent and around 7 percent respectively. In September 2017, the CBU implemented foreign exchange market liberalization to address the dual exchange rate system (official and black market rates). This reform involved unifying the exchange rates by allowing the national currency, the Uzbek soum (UZS), to float more freely. As a result, the official exchange rate was devalued by approximately 100 percent, with the exchange rate against the US dollar shifting from 4,210 UZS/USD to 8,100 UZS/USD (Figure 3). This move aimed to enhance transparency, eliminate market distortions, and align the currency's valuation with market fundamentals, fostering better integration with the global economy.

However, the devaluation caused an immediate surge in inflation, as import prices rose sharply due to the higher exchange rate. By the end of 2017 and the beginning of 2018, inflation rates spiked, driven by increased costs of goods and spillover effects on services, particularly for imports. Despite the short-term inflationary pressures, the reform was viewed as a critical step toward achieving long-term economic stability and improving monetary policy effectiveness.

The overnight interbank interest rate is analyzed for the period from 2006Q2 to 2024Q3, with real interest rates calculated using the moving average of inflation over the previous four quarters (Figure 2). Real interest rates were lower when calculated using headline inflation due to the higher levels of headline inflation compared to core inflation. The sharp increase in inflation observed in late 2017 and early 2018, following the foreign exchange market liberalization and subsequent devaluation of the national currency, was also reflected in the real interest rates. During this period, real rates turned negative under both core and headline inflation measures, highlighting the inflationary impact of the currency devaluation.

The real effective exchange rate (REER) is calculated using the exchange rate and inflation data of 14 key trading partners and the Eurozone, weighted by their respective shares in Uzbekistan's

Figure 2: Overnight rates and real rates

trade. The REER provides a comprehensive measure of the competitiveness of the national currency by accounting for both nominal exchange rate movements and relative price levels. The base year for the REER calculation is set at the beginning of 2006 (Figure 3), establishing a benchmark for comparisons over time. This metric is particularly important for assessing shifts in external competitiveness and understanding the impact of exchange rate dynamics on trade and inflation. A higher REER indicates potential loss of competitiveness due to stronger domestic currency or higher inflation, while a lower REER suggests improved competitiveness.

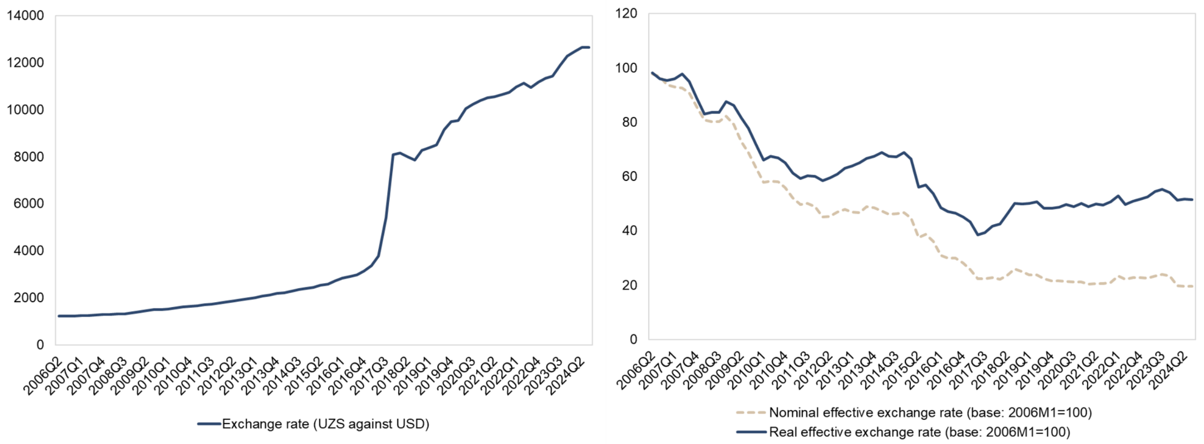

Figure 3: Exchange rates and REER/NEER

For the QPM of the CBU, headline inflation is used for calculating the real interest rate. This choice aligns with the CBU's IT framework, which prioritizes headline inflation due to its accessibility and comprehensibility for the public. Any changes in headline inflation are carefully considered in the monetary policy decision-making process. Furthermore, the QPM incorporates an extensive range of data series, including remittances, loans, fiscal data, foreign direct investment, and other relevant economic indicators. Detailed information on the specific data utilized for each model component, along with their respective sources, is provided in Appendix 1.

4.2 Estimation methodology

In this research, Bayesian estimation is employed to estimate the parameters of the model, combining prior beliefs with observed data. The posterior distribution, which represents the updated belief about the parameters after observing the data, is computed using Bayes' theorem:

$$p(\theta \mid \mathbf{y}) = \frac{p(\mathbf{y} \mid \theta)p(\theta)}{p(\mathbf{y})}$$

Here, $p(\theta \mid \mathbf{y})$ is the posterior distribution, $p(\mathbf{y} \mid \theta)$ is the likelihood of the observed data given the parameters, $p(\theta)$ represents prior beliefs about the parameters, and $p(\mathbf{y})$ is the marginal likelihood ensuring proper normalization.

Given the complexity of the model, the posterior is often analytically intractable. Thus, numerical methods such as Markov Chain Monte Carlo (MCMC) are utilized to approximate the posterior distribution. In cases where conjugate priors are assumed, posterior distributions are derived analytically for efficiency.

This methodology enables a systematic incorporation of prior knowledge while allowing the data to guide the estimation process, offering a robust framework for parameter inference.

The prior means and standard deviations of the parameters were determined through ordinary least squares (OLS) estimation in the equations of the semi-structural models. All parameters were assumed to follow normal distributions, with the initial hyperparameters derived from these OLS estimates. For the smoothing parameter in the Taylor rule, ρ_{ip}, a Beta distribution was employed, as it is constrained to the interval $[0, 1]$.

The coefficients of the QPM model are calibrated – a practice widely adopted among central banks. Calibration is informed by the stylized facts about the Uzbek economy and peer-economies. It also helps in dealing with short data samples and frequent structural breaks. The model is further calibrated to match desired characteristics, such as impulse response functions, sacrifice ratio or exchange rate pass-through.

5. Estimation results

5.1 Semi-structural Model 1 (HLW-type)

The estimation of the natural rate of interest for Uzbekistan using the HLW-type semi-structural model shows that the natural rate remained relatively stable throughout the observed period, primarily driven by stable growth and core inflation dynamics, which did not exhibit a strong upward or downward trend. Its maximum value was 4.98 percent, observed early in the sample, while its minimum was 4.16 percent, occurring around 2019. The average value during this adjusted period was approximately 4.5 percent. This decline aligns with long-term trends such as reduced potential growth and evolving saving and investment dynamics.

Variable	Prior Mean	Posterior Mean	90% HPD (Lower)	Interval (Upper)	Prior	Pstdev
a_1	0.435	0.404	0.263	0.553	Norm	0.100
a_2	-0.030	-0.050	-0.188	0.101	Norm	0.100
a_3	-0.047	-0.011	-0.387	0.372	Norm	0.500
b	0.333	0.329	0.159	0.490	Norm	0.100
c_1	0.571	0.587	0.467	0.711	Norm	0.100
c_3	0.311	0.228	0.088	0.382	Norm	0.100

Table 1: Summary of Priors and Posterior Estimates

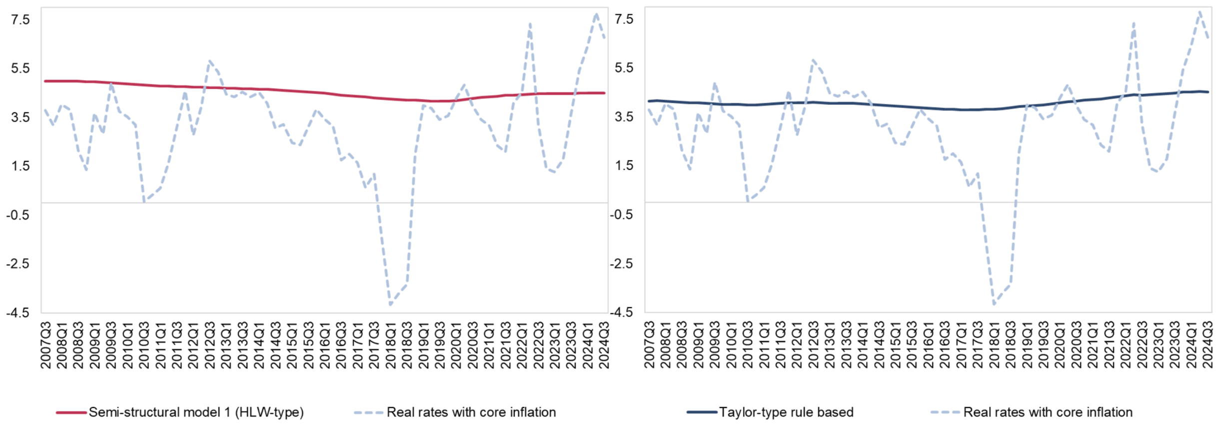

Figure 4: Natural rate of interest estimation

The real interest rate, in contrast, exhibited significant volatility over the analyzed period. Its maximum value reached 7.78 percent, indicative of periods characterized by restrictive monetary policy or high economic returns, while its minimum fell to -4.16 percent during the liberalization of the foreign exchange (FX) market, which coincided with high inflation and low nominal interest rates.

In preparation for the transition to an IT regime, CBU raised its policy rate from 9 percent to 14 percent in 2017 and further to 16 percent in 2018. Following the commencement of the active phase of IT implementation in 2020, the CBU adopted the policy rate as its primary monetary policy instrument, maintaining high rates in a tightening stance. However, in response to the economic challenges posed by the COVID-19 pandemic, the CBU reduced the policy rate from 16 percent to 14 percent in 2020 to support the economy. Relative to the estimated natural rate, this policy stance was accommodative. By the end of 2021, the monetary policy stance had become natural as inflationary pressures subsided.

In March 2022, the onset of the Russia-Ukraine war brought heightened inflationary and depreciation pressures, prompting the CBU to increase the policy rate from 14 percent to 17 percent in a tightening move. During 2022, the anticipated adverse effects of the war were less pronounced, leading the CBU to reduce the policy rate to 15 percent in the summer of that year. This shift turned the policy stance accommodative, driven by a surge in remittance inflows, which were twice as high as in 2021. These inflows bolstered aggregate demand, thereby contributing to inflationary pressures.

In March 2023, the CBU further cut the policy rate to 14 percent, maintaining this level until July 2024. As of July 2024, the policy rate stands at 13.5 percent, which, compared to core inflation, reflects a tight policy stance. This tightening is partially attributable to energy price hikes driven by liberalization in the sector, which increased headline inflation in 2024.

Overall, the CBU maintained a prolonged period of low interest rates before transitioning to the IT regime. Following the active phase of IT implementation, the bank adhered to a rule-based approach and utilized the policy rate as its main instrument. Nevertheless, external and domestic shocks, such as the COVID-19 pandemic and the Russia-Ukraine conflicts, disrupted the CBU's ability to maintain consistently tight monetary conditions, delaying progress toward its inflation target.

5.2 Semi-structural Model 2 (modified HLW-type model)

Variable	Prior Mean	Posterior Mean	90% HPD (Lower)	Interval (Upper)	Prior	Pstdev
a_1	0.420	0.375	0.224	0.536	Norm	0.100
a_2	-0.039	-0.059	-0.208	0.080	Norm	0.100
a_3	-0.042	0.140	-0.299	0.526	Norm	0.500
a_4	-0.012	-0.022	-0.127	0.077	Norm	0.500
b	0.333	0.308	0.145	0.464	Norm	0.100
c_1	0.571	0.569	0.451	0.701	Norm	0.100
c_3	0.311	0.242	0.089	0.397	Norm	0.100
ρ_{ip}	0.534	0.784	0.707	0.862	Beta	0.100
β	1.250	0.902	0.645	1.167	Norm	0.200
θ	0.100	0.129	0.000	0.295	Norm	0.200
ϕ	-0.100	0.001	-0.091	0.089	Norm	0.200
b_1	1.137	1.285	1.195	1.370	Norm	0.100
b_2	-0.320	-0.312	-0.399	-0.220	Norm	0.100

Table 2: Summary of Priors and Posterior Estimates

The estimation of the natural rate of interest using a Taylor-type rule-based semi-structural model indicates that the natural rate remained relatively stable throughout the observed period. However, it was consistently lower from the beginning of the sample period until 2017, by approximately 0.7 percentage points, compared to the HLW-type model (semi-structural Model 1). This difference gradually diminished and effectively closed by early 2019.

The natural rate reached a maximum of 4.5 percent early in the sample and a minimum of 3.8 percent around 2017, with an average value of approximately 4.08 percent during the adjusted period. These results highlight the relative stability of the natural rate, despite structural and cyclical factors that may have influenced interest rate dynamics. This pattern suggests that the CBU maintained an accommodative monetary policy stance. The central focus before 2017 was ensuring the stability of the national currency.

An important shift occurred in 2017 with the introduction of foreign exchange (FX) liberalization reforms. This policy move led to a significant depreciation of the national currency, resulting in inflationary pressures. Despite these shocks, the Taylor-type rule natural rate remained relatively stable at approximately 3.8 percent, while real rates plunged into negative territory, reaching -4.16 percent in 2018Q1. This reflects a natural-to-loose monetary policy stance during the early transition phase when the CBU prioritized currency stability over controlling inflation. By 2019, real rates turned positive again, rising to 3.98 percent in 2019Q1, signaling a gradual normalization of monetary policy as inflationary pressures moderated.

The most pronounced changes in monetary policy occurred during the inflation-targeting (IT) regime introduced in 2020. This period marked a decisive shift toward tightening monetary policy to anchor inflation expectations and enhance price stability. Taylor-type rule rates rose steadily from 4.06 percent in 2020Q1 to 4.53 percent by 2024Q2, while real interest rates surged from 4.28 percent to a peak of 7.78 percent in the same period.

The overall trajectory of the CBU's policy stance highlights the trade-offs it faced during critical transitions. While the pre-2017 period prioritized stable growth and currency stability, the FX liberalization reforms in 2017 created short-term challenges as inflation surged. The policy response, although cautious initially, became progressively aligned with the inflation-targeting framework after 2020. This evolution underscores the CBU's efforts to modernize its monetary policy framework while navigating structural reforms and external shocks.

Numerically, the results confirm this policy evolution. Taylor-type rule rates remained steady through 2007–2016 but declined marginally, signaling accommodation. However, during the FX liberalization phase (2017–2019), a divergence between real and Taylor-rule rates reflected temporary misalignment as the policy focus shifted. The post-2020 period saw a convergence of Taylor-rule rates and real rates, reinforcing the tightening stance required for IT.

5.3 Semi-structural model 3 (QPM model of CBU)

The estimation of the natural rate of interest using the Quarterly Projection Model reveals a dynamic pattern of the policy stance of the CBU over the observed period (Figure 5). The analysis highlights distinct phases of monetary policy, particularly in light of the adoption of the IT regime and the introduction of the key policy rate as the main policy instrument starting in 2020.

During the early part of the sample (2007–2016), the real interest rate was generally below the estimated natural rate, indicating an accommodative policy stance. This suggests that monetary policy during this period was aimed at stimulating economic activity. The gap between the real and natural rates widened considerably between 2016 and 2018, reaching its peak negative values in 2017Q4 at -8.6 percentage points. As mentioned, this significant divergence reflects Inflationary pressure because of FX market liberalization.

From 2018 onward, a gradual tightening of monetary policy is observed, with the real interest rate converging towards the natural rate. By 2019, the gap had narrowed considerably, indicating a shift towards a more natural stance. This period aligns with preparatory measures by the CBU in anticipation of the IT regime.

Following the official adoption of the IT regime in 2020, the real interest rate consistently exceeded the natural rate, marking a transition to a contractionary policy stance. This shift reflects the CBU's commitment to controlling inflation and anchoring inflation expectations. The real interest rate peaked at 6.31 percent in 2023Q3, significantly above the natural rate of 2.97 percent, highlighting the central bank's decisive response to inflationary pressures during this period.

The average real interest rate during the IT regime period (2020–2024) was approximately 4.67 percent, compared to an average natural rate of 3.02 percent. This sustained positive gap underscores the CBU's focus on price stability, in line with its monetary policy objectives.

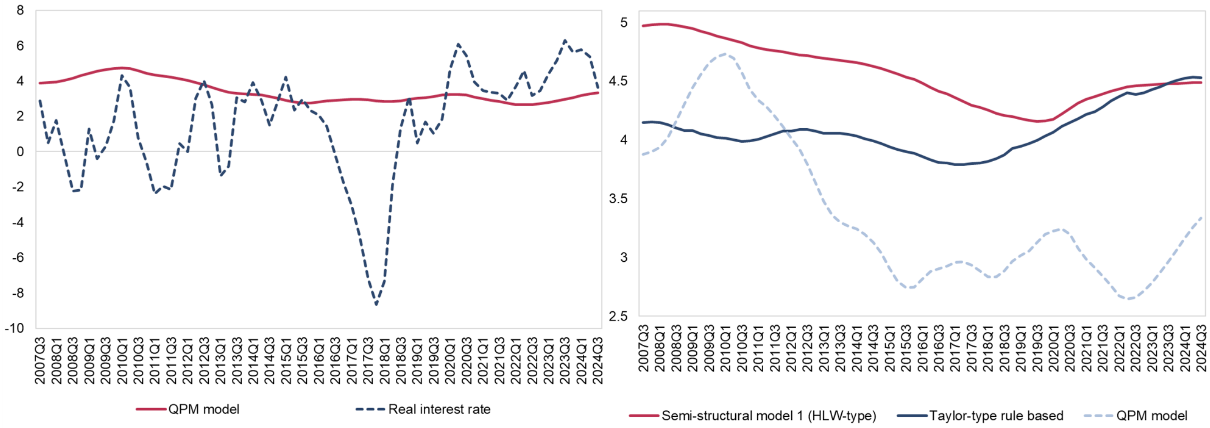

Figure 5: Natural rate of interest estimation

Overall, across all three models, there is a slightly downward trend in the estimated natural rate of interest for Uzbekistan from 2007Q3 to 2024Q3. The estimates from the HLW-type and modified HLW-type models are relatively close to each other, with minor fluctuations around 4.5 percent. In contrast, QPM models provide lower estimates, particularly during the later years.

The downward trend observed in the QPM model is largely attributed to the global decline in natural interest rates, which reached historically low levels during the period from 2014 to 2020. The global low interest rate environment, driven by factors such as low inflation, demographic shifts, and monetary policies in major economies (e.g., ultra-low rates in advanced economies), contributed significantly to the reduction in the natural rate of interest in Uzbekistan. Additionally, the premium associated with Uzbekistan slightly decreased after the implementation of market-based economic reforms starting in 2017. These reforms, which included exchange rate liberalization, financial sector improvements, and greater integration with global markets, helped lower the country risk premium and aligned Uzbekistan more closely with international market dynamics.

Furthermore, the real equilibrium exchange rate showed a trend of appreciation during this period, reflecting improvements in the domestic economy and a more favorable external balance. This appreciation also contributed to the moderation of the natural rate of interest, as a stronger exchange rate helped reduce inflationary pressures, supporting a lower equilibrium interest rate.

5.4 Comparison with the Federal Reserve's r^* Estimate

The Federal Reserve's estimates of the natural rate of interest r^* have consistently displayed a flat trajectory (average one percent) from global financial crisis 2008 (Figure 6). According to Holston, Laubach, and Williams (2017), the U.S. r^* has stabilized at historically low levels, reflecting structural changes such as declining productivity growth, demographic shifts, and heightened global savings preferences. This flat trend contrasts with the more dynamic estimates observed in Uzbekistan, likely due to the different stages of economic development and the significant influence of structural reforms.

One critical insight from the Federal Reserve experience is the substantial uncertainty associated with the estimates r^*. The unobservable nature of r^*, compounded by the reliance on various model assumptions, leads to wide confidence intervals. This uncertainty underscores the challenges of precisely estimating r^* and its application in policy-making. For Uzbekistan, this insight reinforces the need for cautious interpretation of r^* and a flexible monetary policy framework that can adapt to evolving economic conditions.

Figure 6: Natural rate of interest for US (HLW estimation)

5.5 Prospective Policy Implications of r^* for Uzbekistan

The estimates of r^* presented in this study provide a foundational benchmark for evaluating Uzbekistan's monetary policy stance. However, it is important to recognize that the relatively short sample period introduces limitations, particularly in capturing long-term structural trends and economic cycles. Despite these constraints, the analysis offers several key insights. With the CBU's transition to an inflation-targeting regime, the natural rate of interest (r^*) serves as a crucial anchor for policy decisions. The findings suggest that the policy rate has been generally aligned with or above r^* in recent years, reflecting a contractionary stance aimed at curbing inflation and stabilizing expectations. This alignment is particularly relevant as Uzbekistan seeks to achieve its medium-term inflation target of 5 percent by 2026.

The short sample period encompasses periods of significant structural reforms, such as exchange rate liberalization, and external shocks, including the COVID-19 pandemic and geopolitical tensions. These events have shaped the trajectory of real interest rates and monetary policy decisions. The flat or slightly declining trend in r^* observed in the results underscores the need for a nuanced approach to monetary policy that balances inflation control with supporting economic growth.

Ongoing structural reforms, including efforts to reduce dollarization, strengthen the banking sector, and develop financial markets, are critical to improving the transmission of monetary policy. The persistent gap between the real rate and r^* highlights the importance of continuing these reforms to ensure that changes in the policy rate effectively influence lending, borrowing, and economic activity.

Uzbekistan's r^* estimates align with trends observed in emerging markets, where structural factors, such as higher growth potential and external vulnerabilities, influence the natural rate. Unlike advanced economies with flat or declining r^*, Uzbekistan's estimates reflect ongoing economic transitions. Policymakers should remain attentive to global economic developments, particularly shifts in international capital flows and monetary policy trends in major economies, as these factors can influence r^* through risk premiums and external borrowing costs.

While the exact level of the natural rate of interest varies depending on the model used, the general trend indicates a slightly reduction in the natural rate over time. This pattern may reflect structural changes in Uzbekistan's economy, such as shifts in potential output growth, inflation dynamics, and global economic factors. These results provide important insights for the formulation of monetary policy, as the natural rate of interest is a key determinant of the appropriate policy stance for stabilizing inflation and output.

6. Conclusion

This study provides an estimation of the natural rate of interest for Uzbekistan, utilizing three distinct econometric frameworks: the HLW-type model, a Taylor-rule-based variant, and the CBU's QPM. These methodologies collectively reveal a stable average natural rate of approximately 4.0 percent, with individual model estimates ranging from 3.4 percent to 4.5 percent. The findings underscore the significance of the natural rate as a benchmark for shaping monetary policy, particularly during Uzbekistan's economic transitions.

A critical limitation of this analysis stems from the relatively short estimation period, which constrains the ability to observe significant trends in potential growth and inflation. Consequently, the natural rate estimates exhibit minimal variation over time, reflecting the stable but short-term economic conditions captured within the data. Addressing this limitation requires the inclusion of more informative variables in future studies to improve the robustness of r^* estimation. Secondly, most r^* models, such as the Holston-Laubach-Williams framework, have been developed and calibrated primarily for the United States. These models are inherently more suited to the economic structures and data characteristics of advanced economies. When applied to Uzbekistan, a small open economy undergoing significant transitions, these models may require substantial modification to account for unique structural and external factors. Future research could focus on developing r^* models that better reflect the realities of small economies, emphasizing the integration of domestic reforms and external vulnerabilities into the estimation framework. By addressing these limitations, future work can enhance the accuracy and policy relevance of r^* estimates, ultimately supporting more effective monetary policymaking in Uzbekistan.

Despite this, the results still provide meaningful insights into Uzbekistan's monetary policy stance and its alignment with macroeconomic objectives. The analysis identifies key takeaways regarding Uzbekistan's monetary policy evolution. Before 2017, policy was largely accommodative, prioritizing currency stability and economic growth. The liberalization of the foreign exchange market in 2017 marked a critical turning point, as inflationary pressures necessitated a tighter policy stance. The subsequent transition to IT in 2020 reinforced this shift, with real interest rates consistently exceeding the natural rate, signaling a contractionary stance aimed at price stability.

To achieve Uzbekistan's inflation target of 5 percent by 2026, it remains preferable to maintain tighter monetary conditions. Staying alert is essential, as external shocks and structural rigidities continue to pose risks. A contractionary stance will anchor inflation expectations while supporting the credibility of the IT regime.

In conclusion, this study helps understanding of Uzbekistan's natural rate of interest and its role in monetary policy. Despite limitations in data availability and trends, the findings highlight the importance of continued reform to improve policy transmission and align monetary tools with macroeconomic goals. Future research should explore how these reforms and global economic changes may influence the natural rate and its application in Uzbekistan's evolving policy landscape.

References

- Abrams, R. (2005). The effect of capital accumulation on the natural rate of interest. *Journal of Economic Growth*, 10(2), 131–157.
- Barro, R. J. (1991). Economic Growth in a Cross-Section of Countries. *Quarterly Journal of Economics*, 106(2), 407–443.
- Bernanke, B. S. (2005). The Global Saving Glut and the U.S. Current Account Deficit. *Speech at the Virginia Association of Economists.*
- Blanchard, O. J., Cerutti, E., & Summers, L. H. (2017). Inflation and activity – Two exploratory views. *NBER Working Paper No. 23726.*
- Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, 77(3), 623–685.
- Buncic, D. (2024). Econometric Issues in the Estimation of the Natural Rate of Interest. *Economic Modelling*, 132, 106641.
- Caballero, R. J., Farhi, E., & Gourinchas, P. O. (2008). An equilibrium model of "global imbalances" and low interest rates. *American Economic Review*, 98(1), 358–393.
- Calvo, G. A., Leiderman, L., & Reinhart, C. M. (1993). Capital inflows and real exchange rate appreciation in Latin America: The role of external factors. *IMF Staff Papers*, 40(1), 108–151.
- Gordon, R. J. (2016). *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War.* Princeton University Press.
- Gourinchas, P. O., & Rey, H. (2016). Global Imbalances and Currency Wars at the ZLB. *NBER Working Paper No. 22638.*
- Harvey, A. C. (1989). *Forecasting, Structural Time Series Models, and the Kalman Filter.* Cambridge University Press.
- Hledik, T., & Vlcek, J. (2018). Quantifying the Natural Rate of Interest in a Small Open Economy: The Czech Case. *Working Paper Series*, 7.
- Holston, K., Laubach, T., & Williams, J. C. (2016). Measuring the Natural Rate of Interest: International Trends and Determinants. *Federal Reserve Bank of San Francisco Working Paper*, 2016-11.
- IMF. (2023). *World Economic Outlook: A Rocky Recovery.* Chapter 2: The Natural Rate of Interest: Drivers and Implications for Policy.
- Kose, M. A., Prasad, E., & Taylor, A. M. (2009). Thresholds in the process of international financial integration. *Journal of International Money and Finance*, 28(1), 72–94.
- Laubach, T., & Williams, J. C. (2003). Measuring the Natural Rate of Interest. *Review of Economics and Statistics*, 85(4), 1063–1070.
- Mishkin, F. S. (2007). *Monetary Policy Strategy.* MIT Press.
- Pedersen, J. (2015). The Danish Natural Real Rate of Interest and Secular Stagnation. *Danmarks Nationalbank Working Papers*, 94.
- Rachel, L., & Smith, T. D. (2015). Secular drivers of the global real interest rate. *Bank of England Staff Working Paper No. 571.*
- Taylor, J. B. (1993). Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

- Teodoru, I. R., & Toktonalieva, A. (2020). Estimating the natural Interest Rate in the Kyrgyz Republic. *IMF Working Paper*, WP/20/87.
- Wicksell, K. (1936). *Interest and Prices.* London: Macmillan. (Translation of the 1898 edition by R. F. Kahn).

Appendix 1

Indicators	**Frequency**	**Source**
Interbank rates	Monthly	Central Bank of Uzbekistan
Exchange rate	Monthly	Central Bank of Uzbekistan
Core inflation	Monthly	Statistics Agency
Fruit and vegetable inflation	Monthly	Statistics Agency
Regulated prices	Monthly	Statistics Agency
Loans, stock	Monthly	Central Bank of Uzbekistan
New issued loans	Monthly	Central Bank of Uzbekistan
Non-performing loans	Monthly	Central Bank of Uzbekistan
Remittances	Quarterly	Central Bank of Uzbekistan
Foreign direct investment (FDI)	Quarterly	Central Bank of Uzbekistan
Wages	Quarterly	Statistics Agency
Gross domestic product	Quarterly	Statistics Agency
Government external debt	Quarterly	Ministry of Economy and Finance
Government domestic debt	Quarterly	Ministry of Economy and Finance
Fiscal deficit	Quarterly	Ministry of Economy and Finance
Interest costs for serving debt	Quarterly	Ministry of Economy and Finance
Inflation in trading partners	Quarterly	Global Projection Model Network
Exchange rate in trading partners	Quarterly	Global Projection Model Network
Interest rate in trading partners	Quarterly	Global Projection Model Network
Gold prices	Quarterly	Global Projection Model Network
Oil prices	Quarterly	Global Projection Model Network

Table 3: The data used in QPM model