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An estimated DSGE model with financial accelerator: the case of Tunisia

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Abstract

This paper estimates an open economy DSGE model with financial accelerator à la Bernanke et al. (1999)[2], enriched with wage rigidities and imperfect exchange rate pass through. The objective of this paper is to assess the importance of financial frictions and their role in the transmission of transitory shocks in the Tunisian Economy.

The model is estimated by Bayesian technics via Metropolis Hasting algorithm. Using Tunisian data, we obtain an estimate for the external risk premium, indicating the importance of the financial accelerator and the potential balance sheet vulnerabilities for macroeconomic fluctuations. Furthermore, results of the impulse responses functions model support that the inclusion of the financial accelerator magnifies the impact of shocks thereby increasing real fluctuations.

Key words: DSGE, Financial frictions, Bayesian estimation.

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[2] Bernanke et al.[Bernanke,B., M.Gertler, M.,S.Gilchrist,.1999:"The financial accelerator in a quantitative business cycle framework" in Handbook of Macroeconomics, vol.1 c,ed.by Taylor and M.Woodford. Amsterdam].

1. Introduction

The actual economic conditions in Tunisia are characterized by a deceleration of credit, of the GDP growth rate and investment. These features are in conjunction with striking drop in the global refinancing conditions while there is a high level of monetary base and high inflation. The regular refinancing committee of the Central Bank of Tunisia discusses recent trends in the growth of business lending and analyses interest rate spreads. These discussions reflect a view of the important role of financial frictions to amplify the magnitude and the persistence of fluctuations in economic activity.

Many empirical studies as Christensen, I., and Dib, A. (2008), show that the balance sheets conditions of borrowers influence the business cycle through their impact on the external finance premium. In fact, lenders cannot perfectly observe and control the risk involved in supplying funds to borrowers and therefore require compensation of expected losses. Therefore, economic slowdown is characterized by a drop in firm's net worth which reduces their ability to borrow (since they have to pay a higher premium), which in turn slowdown the economic activity even further. The presence of this so called "financial accelerator mechanism" is then explained by the feedback effects of the shocks that affect the debt to equity ratio- leverage of firms which affect their ability to lend. The more leveraged they are the larger are the impact losses on the reduction in lending.

Confronted to the increase in their financing cost forces the firms to reduce their demand for loans and therefore they cut back investment and increase the utilization rate of capital. Both investment and output decline. Subdued aggregate demand feeds back to banking sector resulting in lower profit. This in turn, causes financial intermediaries to further tighten credit supply and raise lending spreads in order to satisfy their balance sheet constraint. Given the decline in lending volume, financial intermediaries can only try to increase profit by increasing spreads, which is likely to lead a further fall in lending demand.

Based on earlier work by Bernanke, Gertler and Gilchrist (BGG (1999)), which develop the links between the borrowing costs of firms and their net worth, we develop and estimate a Dynamic Stochastic General Equilibrium (DSGE) model which sets a micro financial contracting problem between borrowers and lenders proposed by BGG enriched with wage rigidities and imperfect exchange rate pass through. This model distinguishes households and firms in order to explicitly motivate lending and borrowing. In addition, the external finance premium depends inversely on the value of entrepreneurs own net worth.

In this paper, we evaluate empirically the importance of the financial accelerator in the amplification of initial shocks to the Tunisian economy. To do this, we compare two versions of the models: a model that includes a financial accelerator and a model without it.

We estimate the main parameters of the model by using Bayesian approach via Metropolis-Hasting algorithm. This strategy provides a complete characterization of uncertainty in estimating structural parameters by simulating posterior distributions.

Our main finding is that the estimate of the elasticity of external finance premium with respect to the firm leverage is statically significant and close to values used in calibrations for many studies. Results of the impulses responses functions show that introducing the financial accelerator in the model, magnifies the effects of monetary policy shocks on investment and output. This highlights the important role of the financial accelerator in the transmission of monetary policy shocks.

These results are consistent to the findings in Rasaki, Mutiu et al. (2017) who find evidence of balance sheet effects in African economies. Elekdag et al. (2006) finds that balance sheet vulnerabilities magnified the impact of shocks during the Korean crisis. As Christensen, A.Dib (2008), who finds that external finance premium, is statistically significant and away from zero in USA from 1979Q3 through 2004Q3. Also, the model with a financial accelerator generates business cycle moments which are closest to the observed data.

This paper is organized as follows. Section 2 describes the model. Section 3 describes the data and econometric method used to estimate the model. Section 4 shows the empirical results and section 5 concludes.

2. Model

This section presents the main features of the BGG model for a small open economy adding wage rigidities and imperfect exchange rate pass through. The economy is populated by representative household, entrepreneurs, policy makers (central bank and government) and the foreign sector. Entrepreneurs borrow from a financial intermediary that converts households' deposits into business financing for the purchase capital. The presence of asymmetric information between entrepreneurs and lenders creates financial frictions that make the entrepreneurial demand of capital depending on their financial position.

2.1 Entrepreneurs: Financial frictions

The banks serve as intermediaries of financial funds: They attract deposits from domestic households and channel these funds toward entrepreneurs in form one-period standard nominal debt contract (loans) which specify both the amount borrowed and the interest rate to be paid. All depositors receive a non –state-contingent interest rate, while the banking sector is perfectly competitive and risk-free, as we assume each bank to a large number of agents such that individual entrepreneur's uncertainty cancels out across all loans (so banks don't play an active role in the model).

We assume that entrepreneur's debt is dominated in domestic currency, the probability that an entrepreneur will survive until next period is v, so the expected lifetime horizon is $\frac{1}{1-v}$. This assumption ensures that entrepreneur's net worth (the firm equity) will never be enough to fully finance the new capital acquisition, so they issue debt contracts to finance their desired investment expenditures in excess of net worth.

Under an asymmetric information problem, Bernanke et al (1999) show that -given parameter values associated with the cost of monitoring the borrowers- the financial contract[3] between entrepreneurs and financial intermediaries implies an external

[3] BGG. (1999) derive an optimal contract between entrepreneurs and financial intermediaries under an asymmetric information problem.

finance premium, $S(\frac{n_{t+1}}{q_t K_{t+1}})$, that depends on the size of the borrowers leverage ratio. As $(\frac{n_{t+1}}{q_t K_{t+1}})$ [4] falls, the entrepreneur relies on uncollateralized borrowing (higher leverage) to a larger extent to fund his project. Since this increases the incentive to misreport the outcome of the project, the loan becomes riskier and the cost of borrowing rises. Then the external finance premium is given by:

$$Sp_t = S(\frac{n_{t+1}}{q_t k_{t+1}}) \quad (1)$$

Entrepreneurs' demand for capital satisfies the following optimality condition:

$$E_t(1+r_{t+1}^k) = S(\frac{n_{t+1}}{q_t K_{t+1}})E_t\left[\frac{(1+i_t)}{\pi_{t+1}}\right] \quad (2)$$

Equation (2) provides the foundation for financial accelerator. It links entrepreneur's financial position to the marginal cost of fund. The BGG model implies that gross finance premium is expressed as function of the aggregate leverage ratio. From equations (1) and (2), we derive the log-linearized equation for the external spread:

$$\mathrm{E}_t\left[rr_{t+1}^K + \pi_{t+1} - i_t\right] = sp_t \quad (3)$$

$$sp_t = \mu(\mathrm{q}_t + k_t - n_t) \quad (4)$$

Where μ represents the elasticity of the external finance premium with respect to a change in the leverage position of entrepreneurs.

The aggregate entrepreneurial net worth evolves according to:

$$n_{t+1} = vv_t + (1-v)g_t \quad (5)$$

Where v_t, the net worth of the surviving entrepreneurs, is carried over from the previous period, $(1-v)$ is the share of new entrepreneur entering and g_t (which is exogenous in the model) are the transfers from exiting to newly entering entrepreneurs. Notice that v_t is given by:

[4] The inverse relationship of the external finance premium and the financial conditions of the borrowers.

$$v_t = \left[(1+r_t^k) q_{t-1} K_t - S\left(\frac{\eta_{t+1}}{q_{t-1} K_{t+1}}\right) \left[\frac{(1+i_{t-1})}{\pi_t} \right] (q_{t-1} K_t - n_t) \right] \quad (6)$$

As equations (5) and (6) suggest, the principle source of movements in net worth stems from unanticipated movements in returns and borrowing costs. In this regard, unforecastable variations in asset prices, q_t, is the main source of fluctuations in $(1+r_t^k)$. On the cost side, unexpected movements in inflation are the major sources of fluctuations in net worth. Entrepreneurs going out of business at time t consume and transfer some funds to new entrepreneurs. Thus the consumption by entrepreneurs is given by:

$$C_t^e = n_t \qquad (7)$$

In a model without financial accelerator mechanism, the leverage ratio is equal to one, because entrepreneurs are able to fully finance the new capital acquisition. Therefore, the external spread is equal to zero and the elasticity of premium of the leverage ratio is equal to zero.

2.2 Households

The economy is populated by a continuum of households of measure unity. The model is described in terms of representative consumer:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t^{\,j}) \qquad (8)$$

Where C_t is a composite composition index, $L_t^{\,j}$ is a labour supply and P_t is the consumer price index. The functional form is given by:

$$u = \frac{1}{1-\sigma}(C_t - hC_{t-1})^{1-\sigma} - \chi_t \frac{L_{j,t}^{\,1+\sigma_l}}{1+\sigma_l} \qquad (9)$$

Where χ measure the relative weight of labour argument and σ_l is the inverse Frish elasticity of labour supply.

Where h measures the coefficients of habit in consumption. When $h=0$, the internal habit stock is assumed not be dependent on aggregate consumption.

Composite consumption is a CES function of consumption of home goods and import goods:

$$C_t = \left[a^{1/\rho} C_{H,j,t}^{1-1/\rho} + (1-a)^{1/\rho} C_{H,j,t}^{*\ 1-1/\rho} \right]^{\rho/\rho-1} \quad \rho \geq 0 \qquad (10)$$

The implied consumer prices:

$$P_t = \left[aP_{H,t}^{\ 1-\rho} + (1-a)P_{m,T} \right]^{1/1-\rho} \qquad (11)$$

Consumers maximize their utility subject to the following budget constraint:

$$P_t C_t + D_t = W_t L_{j,t} + \Pi_t + (1+i_{t-1})D_{t-1} + T_t \qquad (12)$$

Where D_t is deposits held at intermediary financial, $W_{j,t}$ is the real wages, T_t are lump sum of taxes and Π_t is dividends received from firms.

The Households choose home goods and imported goods to minimize expenditures on total composite C_t. Demand for home goods and imported goods:

$$C_{Ht} = a\left(\frac{P_{Ht}}{P_t}\right)^{-\rho} C_t \ ; \ C_{Ht}^* = (1-a)\left(\frac{p_{mt}}{p_t}\right)^{-\rho} C_t \qquad (13)$$

The household chooses the paths of $\{C_t, L_t, D_t\}$ to maximize expected lifetime utility equation (8) to the constraint (12):

$$\chi_t \frac{L_t^j}{C_t^{-\sigma}} = \frac{W_t}{P_t} \qquad (14).$$

$$C_t^{-\sigma}(1-h) = \beta(1+i_t)E_t\left[C_{t+1}^{\ -\sigma} \frac{P_t}{P_{t+1}} \right] \qquad (15)$$

The linearization derives the Euler equation:

$$c_t = -\sigma\frac{(1-h)}{1+h}(i_t - E_t\pi_{t+1}) + \frac{1}{1+h}E_t c_{t+1} + \frac{h}{1+h}c_{t-1} \qquad (16)$$

This equation is a standard consumption equation showing that consumption depends on past, expected future consumption and real interest rate.

2.3 Labour supply decisions and the wage setting equation

Households act as price setters in the labour market. Wages are assumed to be adjusted after some random signal that's received with probability $(1-\theta_w)$. A household j will chose wage according to:

$$W_t^j = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\chi_w} W_{t-1}^j \qquad (17)$$

When $\chi_w = 0$, there's no indexation and the wages that cannot be re-optimized remain constant. When $\chi_w = 1$, there's perfect indexation to past inflation.

Households supply differentiated labour services to the wholesale sector, where labour is aggregated according to Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 (L_t^j)^{1/1+\varepsilon_w} d_j\right]^{1+\varepsilon_w} \qquad (18)$$

The optimal demand for labour is:

$$L_t^j = \left(\frac{W_t}{W_J}\right)^{\varepsilon_w+1/\varepsilon_w} L_t \qquad (19)$$

Integrating equation (2) and imposing Dixit-Stiglitz aggregation type function, the aggregate wage is:

$$W_t = \left[\int_0^1 (W_t^j)^{-1/\varepsilon_w\sigma_l} d_j\right]^{-\varepsilon_w} \qquad (20)$$

The law of motion of the aggregate wage index is given by:

$$(W_t)^{-1/\varepsilon_w} = \theta_w \left[W_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\chi_w}\right]^{-1/\varepsilon_w} + (1-\theta_w)(W_t^*)^{-1/\varepsilon_w} \qquad (21)$$

The maximization problem results in the following mark-up equation for the re-optimized wage:

$$\frac{W_t^*}{P_t} = (1+\varepsilon_l\sigma_l)\frac{E_t\left[\sum_{i=0}^{1}(\beta^i\theta_w)L_{t+l_w}^j U_{L,t+l_w}\right]}{E_t\left[\sum_{i=0}^{1}(\beta^i\theta_w)\left(\frac{P_t/P_{t-1}}{P_{t+l_w}/P_{t+lw-1}}\right)^{\chi w} L_{t+l_w}^j U_{C,t+l_w}\right]} \qquad (22)$$

Where U_L the marginal disutility of labour is, U_c is the marginal disutility of consumption. After log-linearizing:

$$\pi_t^w = \chi_w\pi_{t-1} + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\sigma_l)}\left[(mrs_{c,l}-(w_t-p_t)\right] + \beta E_t\left[\pi_{t+1}^w - \chi_w\pi_t\right] \quad (23)$$

Where π_t^w the rate of nominal is wage inflation and $mrs_{c,l}$ is the marginal rate of substitution between leisure and consumption. If real wages are lower than marginal rate of substitution, workers will want to raise their nominal wage when the opportunities to adjust will arise.

2.4 The Firm's production

The production is constant returns to scale, presented as:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \qquad (24)$$

Where Y_t the aggregate is output of wholesale goods, K_t is the aggregate amount of capital purchased by entrepreneurs in period $t-1$, L_t is a labour input and A_t is an exogenous technology parameter.

The optimal condition of firms to choose the quantity of investment inv_t is as follows:

$$E_t\left[q_t - 1 - \zeta_{inv}\left(\frac{inv_t}{K} - \delta\right)\right] = 0 \qquad (25)$$

Let inv_t denote aggregate investment expenditures; the capital stock evolves according to:

$$K_{t+1} = \zeta_{inv}\left(\frac{inv_t}{K_t}\right)K_t + (1-\delta)K_{t-1} \quad (26)$$

The adjustment costs have included permitting a variable price of capital. The idea is to have asset price variability which contributes to volatility in entrepreneurial net worth. Capital adjustments cost slow down the response of investment to different shocks, which directly affect the price of capital. In the absence of adjustments costs, the capital price q_t is constant and equal to one.

2.5 Foreign sector

Aggregate demand (C_t^*), aggregate index ($p_{F,t}^*$) and interest rate (R_t^*) for the foreign economy. Following Gertler (2007), the law of one price holds at the wholesale level for foreign transactions.

Prices of exports evolve as follow:

$$p_{H,t}^* = \frac{p_{H,t}}{X_t}$$

And demand for exports:

$$C_{H,t}^* = \left[\gamma_2\left(\frac{P_{H,t}^*}{P_{F,t}^*}\right)^{-\eta} C_t^*\right]^{\rho'} C_{H,t}^{*\;1-\rho'} \quad (27)$$

Where η the elasticity of the substitution between imports and home goods is, γ_2 is the share of imports. The parameter ρ' govern the responsiveness of export demand to change in domestic prices $P_{H,t}$. The real exchange rate is defines as:

$$rer_t = \frac{X_t p_{f,t}^*}{p_t} \quad (28)$$

2.6 Central Bank and Government

We include in our model the Taylor-rule of central bank and the process of the government. The central bank implement interest rate rule in order to achieve specific policy objectives:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t + \phi_{\Delta_e} \Delta e_t) + z_t \quad (29)$$

The monetary authority sets the nominal interest rate tacking into consideration the inflation rate, the output gap and the exchange rate depreciation. The central bank is engaged to smooth interest rate.

In fact, process of government is defined as:

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt} \qquad (30)$$

Where ρ_g is the persistence of government spending.

3. The complete log-linearized model

The complete log-linearized model corporates blocks of aggregate demand, aggregate supply, and foreign sector, evolution of the state variables, and monetary policy rule and shocks processes.

3.1 The base model with financial accelerator

The base model is defined by (38) equations:

B1.1: Aggregate demand for home goods

$$\frac{Y_H}{Y}Y_{H,t} = \left((1-\alpha_C)(\frac{C}{Y}+\frac{C^e}{Y}) + (1-\alpha_I)\frac{I}{Y} \right) d_{H,t} + \frac{G}{Y}g_t + \frac{C_H^*}{Y}c_{H,t}^* \quad (1)$$

B1.2: Domestic and private demand for home goods

$$\left((1-\alpha_C)(\frac{C}{Y}+\frac{C^e}{Y}) + (1-\alpha_I)\frac{I}{Y} \right) d_{H,t} = (1-\alpha_C)\frac{C}{Y}c_t + (1-\alpha_c)\frac{C^e}{Y}c_t^e - (1-\alpha_C)(\frac{C}{Y}+\frac{C^e}{Y})\eta_C(p_{H,t}-p_t)$$

$$+(1-\alpha_I)\frac{I}{Y}(inv_t - \eta_I(p_{I,t}-p_t)) \quad (2)$$

B1.3: Foreign demand for home goods

$$C_{H,t}^* = y_t^* - \eta^*(p_{H,t} - p_t - rer_t) \quad (3)$$

B1.4: Volume of imports

$$\frac{M}{Y}m_t = \alpha_C\frac{C}{Y}c_t + \alpha_C\frac{C^e}{Y}c_t^e - \alpha_C(\frac{C}{Y}+\frac{C^e}{Y})\eta_C(p_{F,t}-p_t) + \alpha_I\frac{I}{Y}(inv_t - \eta_I(p_{F,t}-p_{I,t})) \quad (4)$$

B1.5: Volume of exports

$$\frac{X}{Y}x_t = \frac{C_H^*}{Y}c_{H,T}^* + \frac{Y_{CO}}{Y}Y_{CO,t} \quad (5)$$

B1.6: Export deflator

$$\frac{X}{y}(p_{X,t}-p_t)=\frac{C_H^*}{Y}(p_{H,t}-p_t)+\frac{Y_{co}}{Y}(p_{CO,t}^*-p_t^*+rer_t) \quad (6)$$

B1.7: Definition of the real exchange rate

$$\text{rer}_t=rer_{t-1}+\Delta e_t+\pi_t^*-\pi_t \quad (7)$$

B1.8: Definition of the price of Home goods

$$(p_{H,t}-p_t)=(p_{H,t-1}-p_{t-1})+\pi_{H,t}-\pi_t \quad (8)$$

B1.9: Uncovered interest rate parity condition

$$i_t = i_t^* + E_t\left[\Delta e_{t+1}\right]+\varsigma\, b_t^* (9)$$

B1.10: Balance of payment

$$\frac{B^*}{Y}(rer_t+b_t^*)=\frac{B^*}{Y}(i_{t-1}^*+(1+\varsigma)b_{t-1}^*+rer_t-\pi_t^*)+\frac{M}{Y}(rer_t+m_t)$$
$$+\chi\frac{Y_{co}}{Y}(p_{CO,t}^*-p_t^*+rer_t+Y_{CO,t})-\frac{X}{Y}(p_{X,t}-p_t-x_t) \quad (10)$$

B1.11. Imperfect exchange rate pass-through

$$\pi_{F,t}=\frac{\beta}{1+\beta\chi_F}E_t\left[\pi_{F,t+1}\right]+\frac{\chi_F}{1+\beta\chi_F}\pi_{F,t-1}+\frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F(1+\beta\chi_F)}(rer_t-(p_{F,t}-p_t)) \quad (11)$$

B1.12. Dynamic definition of the price of foreign goods

$$(\text{p}_{F,t}-p_t)=(p_{F,t-1}-p_{t-1})+\pi_{F,t}-\pi_t \quad (12)$$

B1.13. investment goods deflator

$$(p_{I,t}-p_t)=(1-\alpha_I)(p_{H,t}-p_t)+\alpha_I(p_{F,t}-p_t) \quad (13)$$

B1.14. Consumption goods deflator

$$0=(1-\alpha_C)(p_{H,t}-p_t)+\alpha_I(p_{F,t}-p_t) \quad (14)$$

B1.15. GDP definition

$$y_t = \frac{Y_H}{Y} y_{H,t} + \frac{Y_{co}}{Y} y_{co,t}, \quad \text{with } \frac{Y_H}{Y} + \frac{Y_{CO}}{Y} = 1 \qquad (15)$$

B1.16. Modified. Euler equation of consumption

$$c_t = -\sigma \frac{(1-h)}{1+h}(i_t - E_t \pi_{t+1}) + \frac{1}{1+h} E_t c_{t+1} + \frac{h}{1+h} c_{t-1} \qquad (16)$$

B1.17 consumption of entrepreneurs

$$c_t^e = \eta_t \qquad (17)$$

B1.18. spread of real return of capital over the cost of funds depend of the financial position of entrepreneurs

$$\mathrm{E}_t\left[rr_{t+1}^K + \pi_{t+1} - i_t \right] = sp_t \qquad (18)$$

B1.19. Definition of the external spread

$$sp_t = \mu(q_t + k_t - n_t) \qquad (19)$$

B1.20 real return to capital

$$rr_t^k = (1-\varepsilon)(mc_t + y_t - k_{t-1}) + \varepsilon q_t - q_{t-1} \qquad (20)$$

$$\varepsilon = \frac{(1-\delta)}{R^k}$$

B1.21 investment adjust cost

$$q_t - (p_{I,t} - p_t) = \zeta_{INV}(inv_t - inv_{t-1}) - \beta \zeta_{INV} E_t\left[inv_{t+1} - inv_t \right] \qquad (21)$$

B1.22 Aggregate supply of home goods

$$y_{H,t} = a_t + \alpha k_{t-1} + (1-\alpha) l_t \qquad (22)$$

B1.23 Definition of the marginal rate of substitution b/w labor and consumption

$$mrs_t = \sigma_L l_t + \frac{1}{\sigma}\frac{1}{(1-h)} c_t - \frac{1}{\sigma}\frac{h}{(1-h)} c_{t-1} \qquad (23)$$

B1.24 Demand for labour

$$rw_t = mc_t + y_t - l_t \qquad (24)$$

B1.25 Philips curve

$$\pi_{H,t} = \frac{\beta}{1+\beta\chi_H} E_t\left[\pi_{H,t+1}\right] + \frac{\beta}{1+\beta\chi_H}\pi_{H,t-1} + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H(1+\beta\chi_H)}(mc_t - (p_{H,t} - p_t)) \quad (25)$$

B1.26 Capital evolution

$$k_t = \zeta_{INV} + (1-\delta)k_{t-1} \qquad (26)$$

B.1.27. Entrepreneur's net worth evolution

$$\eta_t = \frac{K}{N} rr_t^K - (\frac{K}{N} - 1)(sp_{t-1} + i_{t-1} - \pi_t) + \eta_{t-1} \quad (27)$$

B1.28 Monetary policy rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t + \phi_{\Delta_e} \Delta e_t) + z_t \qquad (28)$$

B1.29 Phillips curve for wages

$$\pi_t^w = \chi_w \pi_{t-1} + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w \sigma_L)}(mrs_t - rw_t) + \beta E_t\left[\pi_{t+1}{}^w - \chi_w \pi_t\right] \qquad (29)$$

B1.30 Definition of inflation wages

$$\pi_t^w = rw_t - rw_{t-1} + \pi_t \qquad (30)$$

B1.31 Government expenditure evolution

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \qquad (31)$$

B1.32 Productivity evolution

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \qquad (32)$$

B1.33 New monetary deviation evolution

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t} \qquad (33)$$

B1.34 Foreign interest rate evolution

$$i_t^* = \rho_{i^*} i_{t-1}^* + \varepsilon_{i^*,t} \qquad (34)$$

B1.35 Foreign inflation evolution

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi^*,t} \qquad (35)$$

B1.36 Commodity price evolution (in real foreign terms)

$$(p_{co,t}^* - p_t^*) = \rho_{pco^*}(p_{co,t-1}^* - p_{t-1}^*) + \varepsilon_{pco^*,t} \qquad (36)$$

B1.37 Commodity production evolution

$$y_{co,t} = \rho_{yco} y_{co,t-1} + \varepsilon_{co,t} \quad (37)$$

B1.38 Foreign demand

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_{y^*,t} \qquad (38)$$

3.2 *Interpretation of equations:*

Equation (B1.1) is the log-linearized version of the resources constraint. The primary determinants of the variation in aggregate expenditures, y_h, are households consumption, c, investment, inv_t, government consumption, g_t, and foreign demand for home goods c_h^*. Of lesser importance is variation in entrepreneurial consumption, c^e.

Households consumption is governed by Euler modified consumption, as given by equation (B1.16). If we assume $h = 0$, we obtain a standard consumption Euler equation, which assumed financial market friction do not impede households behaviour. Since entrepreneurial consumption is a small fixed fraction of aggregate net worth, it varies proportionally with aggregate net worth as equation (B1.17).

Equations (B1.18) and (B1.19) characterize the financial accelerator. In the absence of financial market friction, these relations collapse to $E_t(rr_{t+1}^k) - (i_t - \pi_{t+1}) = 0$. Investment is pushed to the point where the expected return on capital, $E_t(rr_{t+1}^k)$, equals to the cost of funds. While, with the presence of financial friction, the cost of external funds depends on entrepreneur's equity holding, net worth relative to the gross value of capital, $n_t - (q_t + k_t)$, it means that a rise in this ratio reduces the cost of external funds, implying that investment will rise.

4. Estimation methodology

We estimate the model using the Bayesian estimation. This approach has many advantages. First, it formalizes the use of prior empirical knowledge about the parameter of interest. Second, Bayesian inference characterizes the posterior distribution of the parameters; they facilitate an accurate assessment of all the uncertainty surrounding the model. Given the likelihood function and a set of prior distributions, an approximation to the posterior mode of the parameters of interest can be calculated.

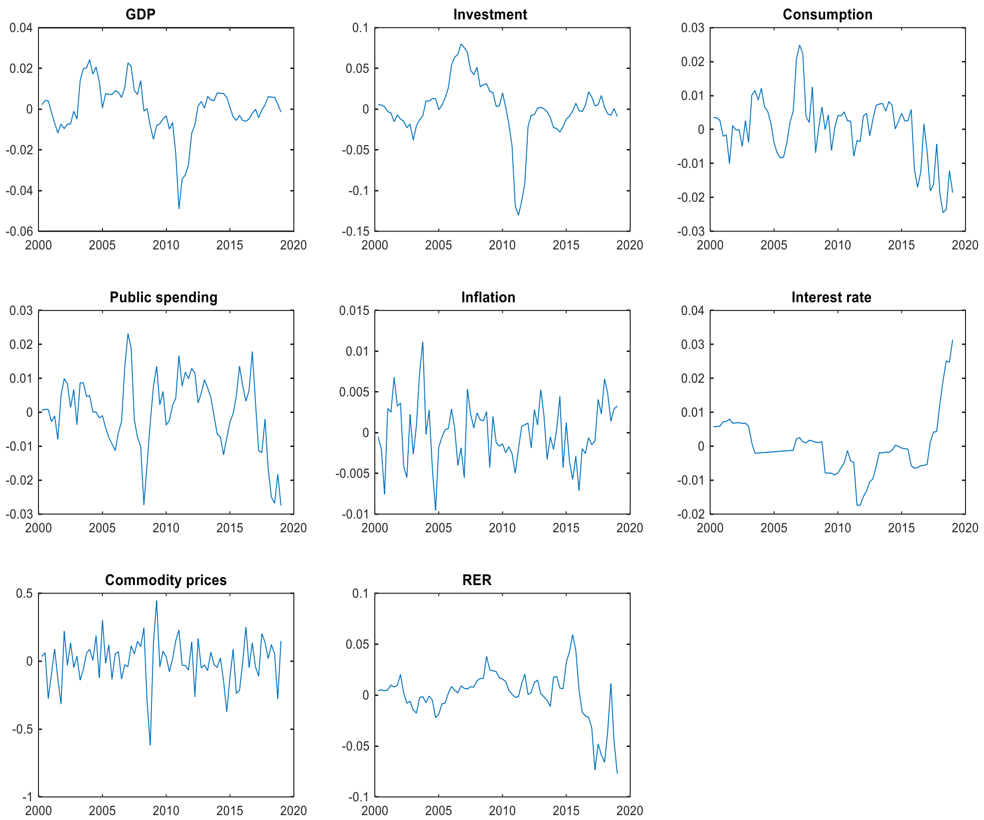
The posterior model is used as staring value for the Metropolis-Hasting algorithm, which allows us to generate draws from the posterior density. At each iteration, a proposal normal distribution with mean equal to the previously accepted draw is used to generate a new draw which is accepted as a draw from the posterior density $p(\theta / x)$ with probability p .The probability p depends on the value of the posterior and the proposal density at the candidate draw, relative to the previously accepted draw. We generate 2000 draws.

In order to ensure that the model is not stochastically singular, we include a number of exogenous shocks. The number of exogenous shocks must be at least large as the number of observed variables.

4.1 Data

To estimate the model we used information of eight key macroeconomic variables for Tunisia running from ***2000Q1 TO 2019Q2***: Real GDP, real investment, real private consumption, real public expenditure, consumer prices index, the nominal interest rate, commodity prices, and real exchange rate. All the variables are expressed using filter of Hodrick-Prescott, with the exception of the nominal interest rate. All the variables are seasonally adjusted using X13, except for the nominal interest rate.

Figure 1: Demeaned variables



4.2 Prior Distributions of the parameters

To avoid identifications problem, we calibrate few parameters. Therefore, the lists of calibrated parameters include the quarterly discount factor, β which set to 0.99. As in much of the literature, the depreciation rate of capital, δ , is set equal at 10 percent per annum, implying a value of the cost share of capital of 0.025. We take the steady state share of government expenditures in total output, G/Y to be 0.2, historical average over our sample. The steady state share of consumption and investment (α_c and α_I) are respectively 0.6 and 0.12.

The inverse of the Frish elasticity is set to 1, as assumed for the inverse of the intertemporal elasticity of substitution in consumption. For the parameters related to the rate price adjustment, we let the probability that a firm does not change its price within a given period θ_H, equal to 0.75, implying that the average period between price adjustments is four quarters.

The parameter ε_L, measures the degree of retailer's monopoly power, is set to implying a steady state price mark-up of 20 percent, a common value used in the literature. The serial correlation parameters for technology and government expenditures are assumed to be 0.95.

In order to be sure about the consistency to be used in the model, not all parameters are free. The assumptions to be used are:

i. A risk spread, $R^k - R$ equal to $0.03[annual]$.
ii. A ratio of capital to net worth ($\frac{K}{N}$) equal to 2 (or equivalently a leverage ratio of 0.5).
iii. The survive rate of entrepreneurs is set to be $\gamma = 0.975$ (quarterly).
iv. We take an idiosyncratic productivity variable $\log(\tilde{\omega})$, to be log normally distributed with variance σ_w^2.

The Parameters which are calibrated are summarized in Table (1):

Table 1: Calibration of parameters

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
β	0.99	ζ_{inv}	20	η^*	0.50	θ_F	0.875
σ	1	$(X-M)/Y$	0.08	ζ	0.001	χ_F	1.0
Y_{CO}/Y	0.05	χ	0.5	θ_H	0.75		
σ_I	1	α_c	0.6	χ_H	0.50		
α	0.34	α_I	0.12	θ_w	0.8125		
δ	0.03	η_c	0.50	χ_w	1.0		
G/Y	0.2	η_I	0.50	ε_L	6		
R^K	$(1/\beta+0.03)^{1/4}$	$Def.rate$	0.0075				

The choice of the priors of the estimated parameters is usually determined by the theoretical implications of the model and evidence from previous studies. In general, inverse gamma distributions are used as priors when non negatively constraints are necessary and Beta distributions for fractions and probabilities. Normal distributions are used when more informative priors seemed to be necessary. In some cases, we use the same priors mean as in previous studies as Elekdag et al. (2006) but we choose a large standard deviations, thus imposing less informative priors and allowing data to determine the parameter's location.

In fact, the parameter, μ is of most interest because is the centre of financial accelerator of the external finance premium. Lacking a reliable benchmark in specifying prior of μ, we choose a Beta distribution for this parameter with a mean 0.07 and standard deviation equal to 0.03.

The parameters describing monetary policy reaction are based on the Taylor rule that allows for interest rate smoothing and is augmented to include responses to the exchange rate in addition to output and inflation. We assume that the central bank smooth interest rate adjustments. Thus we set the prior, ρ_i as a Beta density centred on 0.8 with a standard deviation of 0.20. Priors of the coefficients on inflation, the output gap and the exchange rate, $\phi_\pi, \phi_y, \phi_{dep}$ belong to the Gamma distribution with respectively mean of 1.7, 0.15, 0.1 as well as standard deviations of the monetary policy regimes of 0.1, 0.02, 0.05.

The habit parameter h is assumed to be a Beta distribution with a mean 0.65 and standard deviation 0.1. The priors of the variance of the exogenous stochastic process correspond to an Inverse Gamma distribution with a mean of 0.25 and two degrees of freedom.

4.3 Empirical results

The results of the posterior estimates of each parameter from our model are presented in table 1. Along with the medians, we present the 5th and 95th percentiles of the posterior distributions, which serve to quantify the uncertainty surrounding these estimates. Additional information in our results is presented in Figure (2) that plots kernel density estimates of the posteriors together with priors for subset of the parameters.

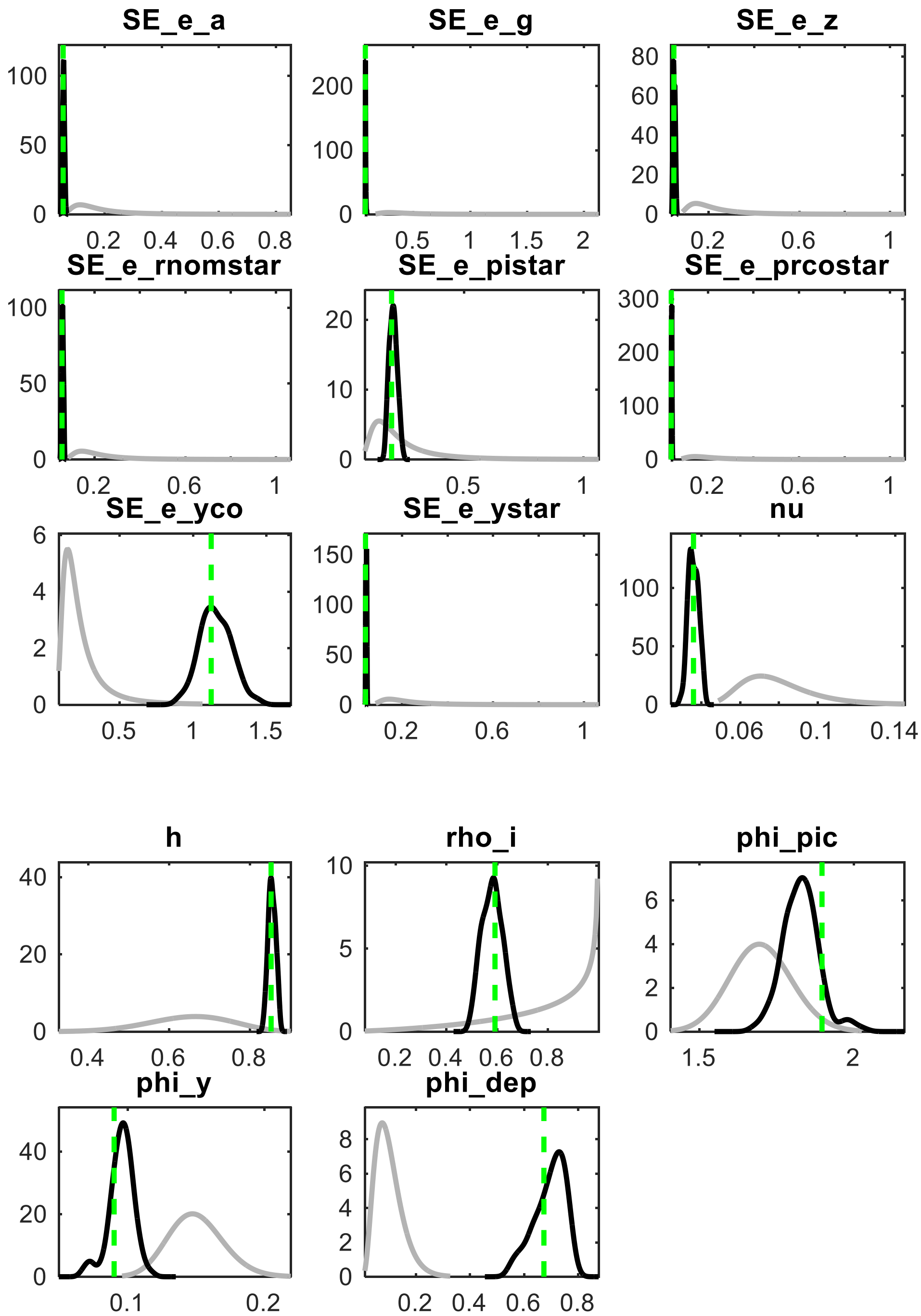
Table2.Prior and Posterior Distribution of parameters

	Prior Distribution			Posterior Distribution		
Parameter	Type	mean	Standard Error	5%	Median	95%
h	*Beta*	0.65	0.1	0.8401	0.86	0.8721
μ	*inverse_Gamma*	0.08	0.02	0.0319	0.04	0.0403
ρ_i	*Beta*	0.8	0.2	0.4952	0.58	0.6548
ϕ_{pic}	*Gamma*	1.7	0.1	1.7905	1.88	1.9686
ϕ_y	*Gamma*	0.15	0.2	0.0775	0.09	0.1044
ϕ_{dep}	*Gamma*	0.10	0.05	0.3849	0.46	0.5286
e_a	*inverse_Gamma*	0.2	0.1540	0.0506	0.0583	0.0655
e_g	*inverse_Gamma*	0.5	0.3225	0.0782	0.0806	0.0840
e_z	*inverse_Gamma*	0.25	0.7454	0.0391	0.0402	0.0418
$e_{rnomstar}$	*inverse_Gamma*	0.25	0.8038	0.1598	0.1815	0.1998
e_{pistar}	*inverse_Gamma*	0.25	1.63	1.0510	1.2032	1.4023
$e_{picostar}$	*inverse_Gamma*	0.25	0.0395	0.0391	0.0407	0.0424
e_{yco}	*inverse_Gamma*	0.25	17.24	3.1428	3.5206	3.8731
e_{ystar}	*inverse_Gamma*	0.25	0.0435	0.1837	0.1916	0.1973

Our empirical results confirm that the data support the inclusion of the financial accelerator in a small open economy as Tunisia. This can be assessed by determining whether the elasticity of the external finance premium, μ , is estimated away from zero, because this would be support the notion that the financial accelerator captures important credit market frictions. In fact, the median estimated of the value of external finance premium is **0.04** per quarter, which implies an annual premium of 1.6 with 5th and 95th percentiles of 0.04 and 0.0403 respectively. This is different from zero despite our prior.

The estimate value of elasticity of the external finance premium with respect to the leverage, which equals to **0.04,** is slightly below the estimate of external finance premium 0.05 for Morocco in Rasaki, Mutiu et al. (2017) and the external premium for South Korea which equals to 0.06 in Elekdag et al. (2006). However, it is higher than estimated value for US post period 1979, which equals to 0.03 in Christensen, A.Dib (2008). Note in the Figure (2), that the posterior is sharply peaked relative to prior distribution, suggesting that the data are quite informative about this parameter. In fact, the parameter μ, could be interpreted as a summary statistic indicating how vulnerable the economy is to shocks affecting aggregate balance sheets.

<u>Figure 2: Priors and Posteriors means</u>



The habit persistence parameter h is estimated at 0.86, implying that consumption features some persistence. This estimation is consistent with findings of many studies as the posterior mean of habit persistence in Morocco, which h equals to 0.74. In Smets and Wouters (2005), the habit parameter for the Euro Area is estimated t0 0.59, relatively low compared to the value estimated for the USA (0.69).

With regards to the estimates of the policy rule parameters, our results indicate that the Central bank of Tunisia places a relatively high weight on controlling the inflation. In fact, the median value of the coefficient on inflation ϕ_{pic} is 1.88, which reflects the authorities 'commitment to price stability. While the estimates of the interest rate response to output gap is low this equals to 0.09, as shown in figure (1). In fact, these estimates suggest that the response of monetary policy is more strongly to exchange rate fluctuations than output, which parameters are estimated respectively, ϕ_{dep}=0.46 and ϕ_y=0.09. This is consistent with results reported by Moez Ben Hassine and Rebeii(2019)[5], who find that the posterior average of the monetary reaction to exchange rate is high than the posterior average of the reaction coefficient to fluctuations of output growth.

[5] Moez Ben Hassine and Nooman Rebeii:"Informality frictions and macroprudential policy" (IMF),2019. These authors used DSGE model to estimate informal sector in the case of Tunisia. Their output posteriors estimates for Taylor rule parameters have justified by the managed exchange policy adopted by the Central Bank of Tunisia.

5. Impulse-responses functions analysis

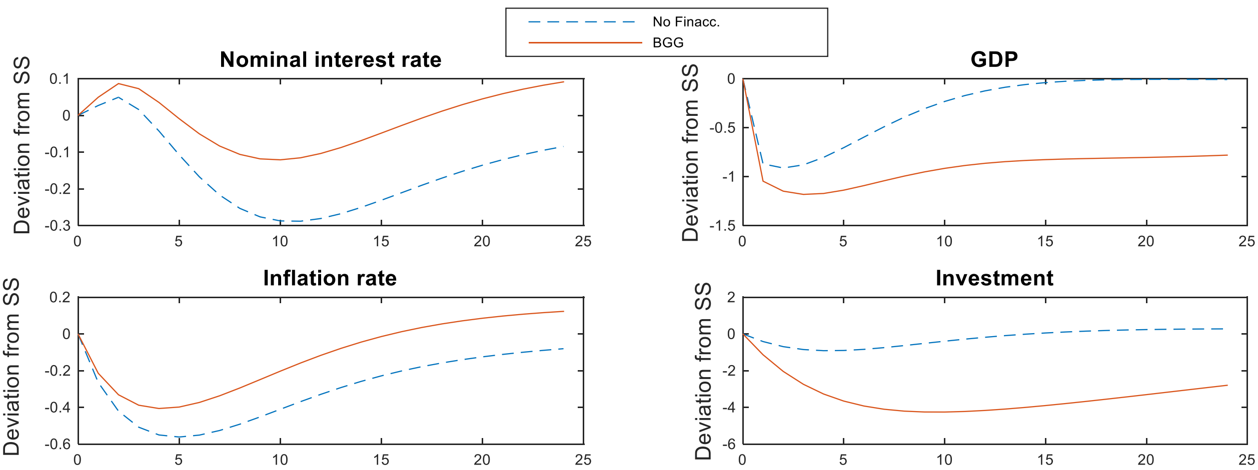
5.1 IRF's model with a Financial Accelerator Vs without a Financial Accelerator

We highlight in this section the impact of some shocks specific to domestic and foreign monetary policies and we compare the impulse responses of key macroeconomic variables to these shocks in the model with a financial accelerator versus a model without a financial accelerator.

These two models are defined as follow:

1. Baseline model-defined with financial accelerator (BGG model).
2. Model without a financial accelerator (BGG model without financial accelerator), by imposing $sp_t = 0$ and $n_t = 0$.

Figure 3: IRF model with and without a financial accelerator to a shock of domestic interest rate



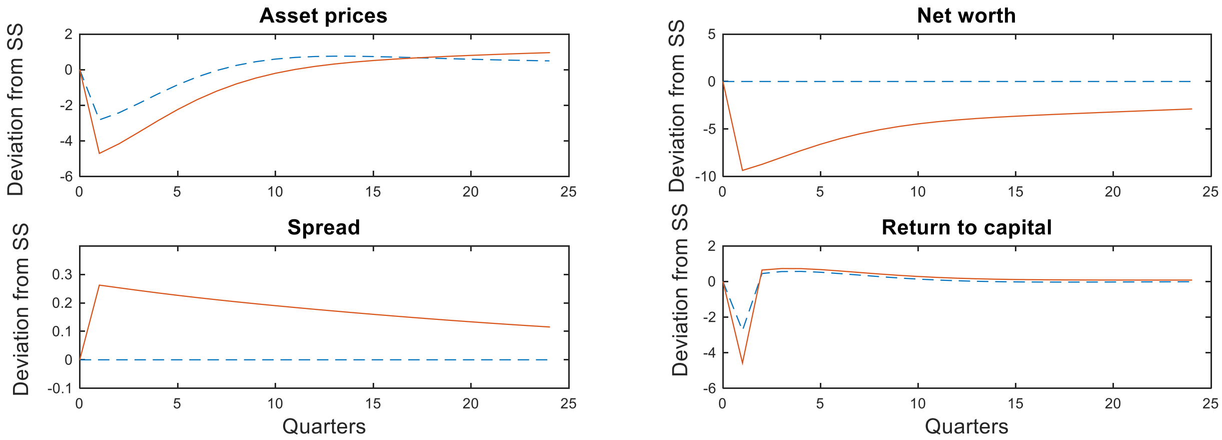


Figure (3) plots the responses to a positive 1% monetary policy shock (tightening). Following the shock, the nominal interest rate rises. The basic mechanism of the financial accelerator is evident in the impulse response. After a tightening monetary policy, net worth falls, because for a given spread, the higher real interest rate today requires a higher real return to capital tomorrow:

$$E_t\left[rr_{t+1}^K+\pi_t-i_t\right]=sp_t$$

But this is only possible if investment today falls, so as to increase tomorrow's marginal product of capital, there should be a lower capital stock tomorrow-i.e there should be a lower capital stock tomorrow.

$$rr_{t+1}^k=(1-\varepsilon)(mc_{t+1}+y_{t+1}-k_t)+\varepsilon q_{t+1}-q_t \; ; \; \varepsilon=\frac{(1-\delta)}{R^k}$$

$$k_t=\delta inv_t+(1-\delta)k_{t-1}$$

Lower investment today is consistent with a reduction in asset prices (i.e tobin's q should fall):

$$q_t=\zeta_{inv}(inv_t-inv_{t-1})-\beta\zeta_{inv}E_t\left[inv_{t+1}-inv_t\right]$$

At the same time, the lower demand for investment goods and the lower consumption, in addition to a lower price of capital, reduces the real rental rate of capital today.

$$rr_{t+1}^k=(1-\varepsilon)(mc_{t+1}+y_{t+1}-k_t)+\varepsilon q_{t+1}-q_t$$

This affects negatively entrepreneur's net worth, which is further reduced by the lower inflation rate:

$$n_t = \frac{K}{N} rr_t^k - (\frac{K}{N} - 1)(sp_{t-1} + i_{t-1} - \pi_t) + n_{t-1}$$

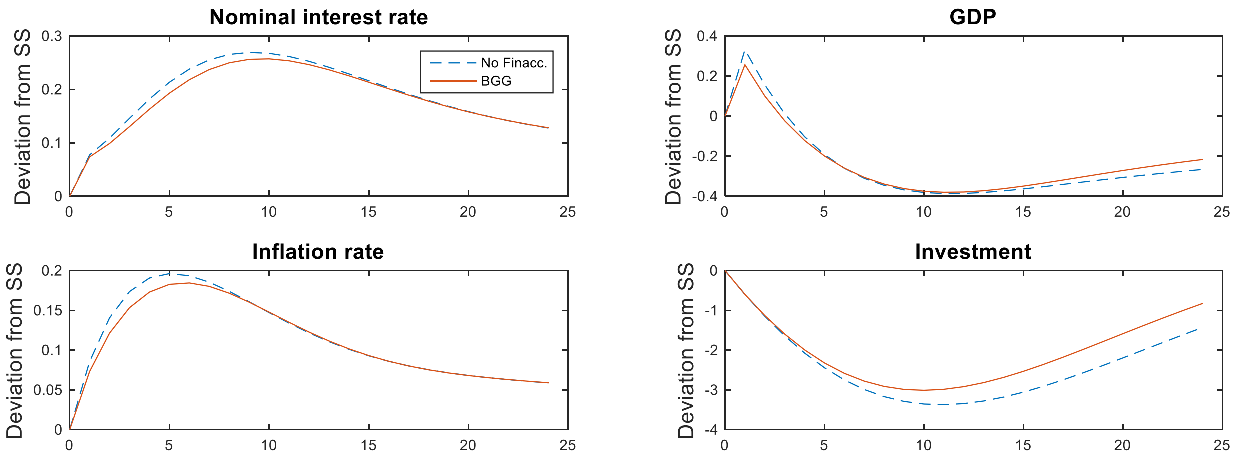
Although there's a fall in asset prices and in next period capital stock, the fall in net worth is relatively large, and thereby the spread finally increases.

$$sp_t = v(q_t + k_t - n_t)$$

Total GDP is also affected by contraction in net worth through lower entrepreneur's consumption. Net worth in the next periods start recovering because the rental rate of capital increases and thus the spread gets lower over time.

Consequently, the presence of a financial accelerator mechanism implies a significant amplification and propagation of the monetary policy shock on investment and capital prices, as the responses of these variables in the presence of financial accelerator model are almost double those in the case of no existence of the financial accelerator.

Figure 4. IRF model with and without financial accelerator due to shock of foreign interest rate shock.



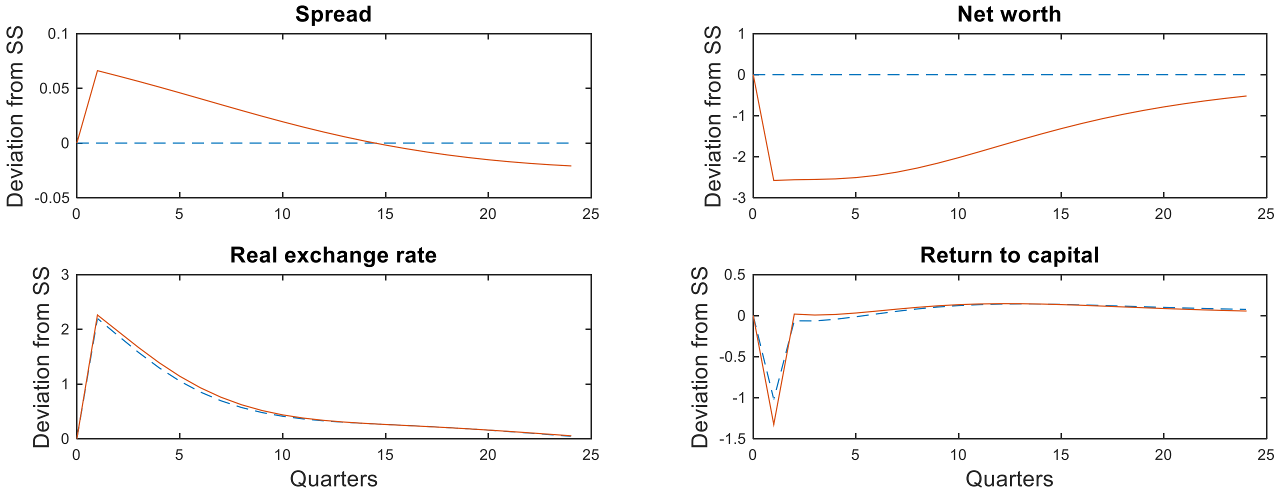


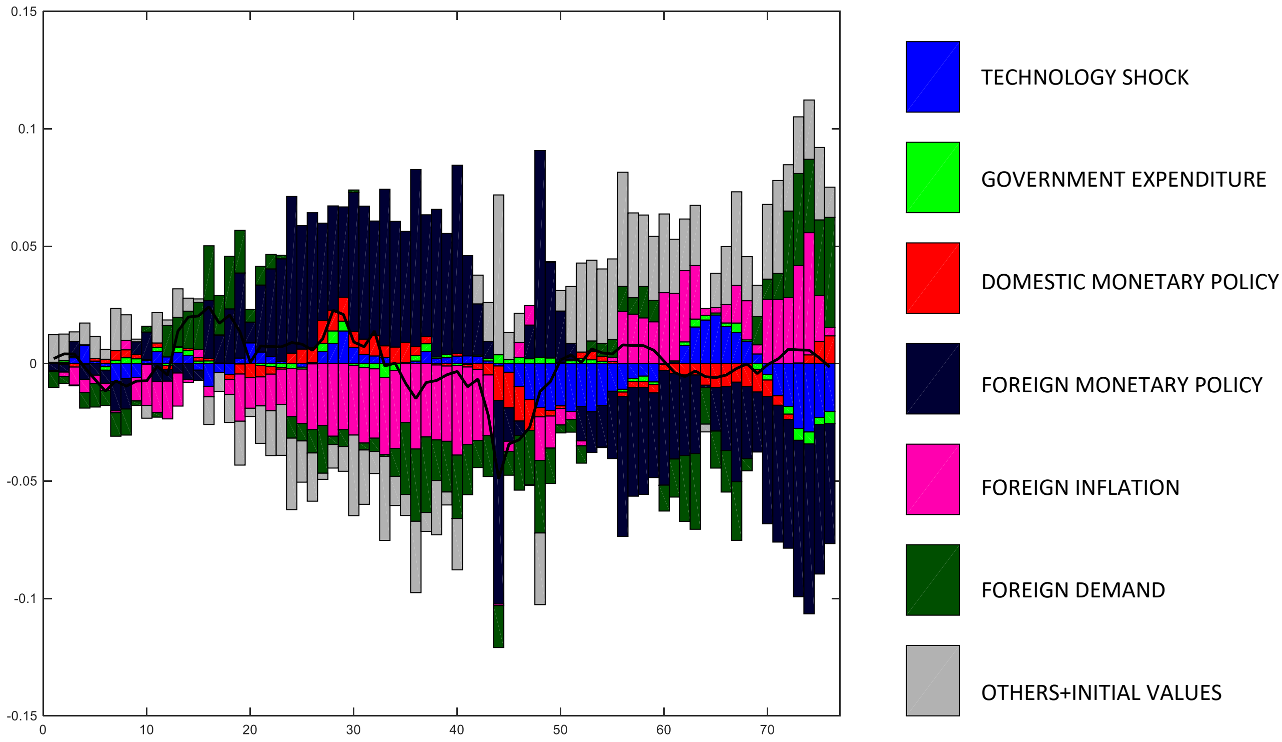
Figure (4), shows that following a 1% positive foreign interest rate , there is a little impact on the output , investment and the return to capital. The depreciation of the exchange rate is little less without financial frictions, but inducing more inflation due to the increase of the marginal costs.

5.2 Variance Decomposition

Starting from the vector moving average representation of the model, any observed or unobserved variable can be broken down into contributions of present and past shock, with weights assigned to previous innovations decaying in accordance to their moment of occurrence. Figure (5), shows the fractions of the forecast error variance of output accounted for by the structural shocks.

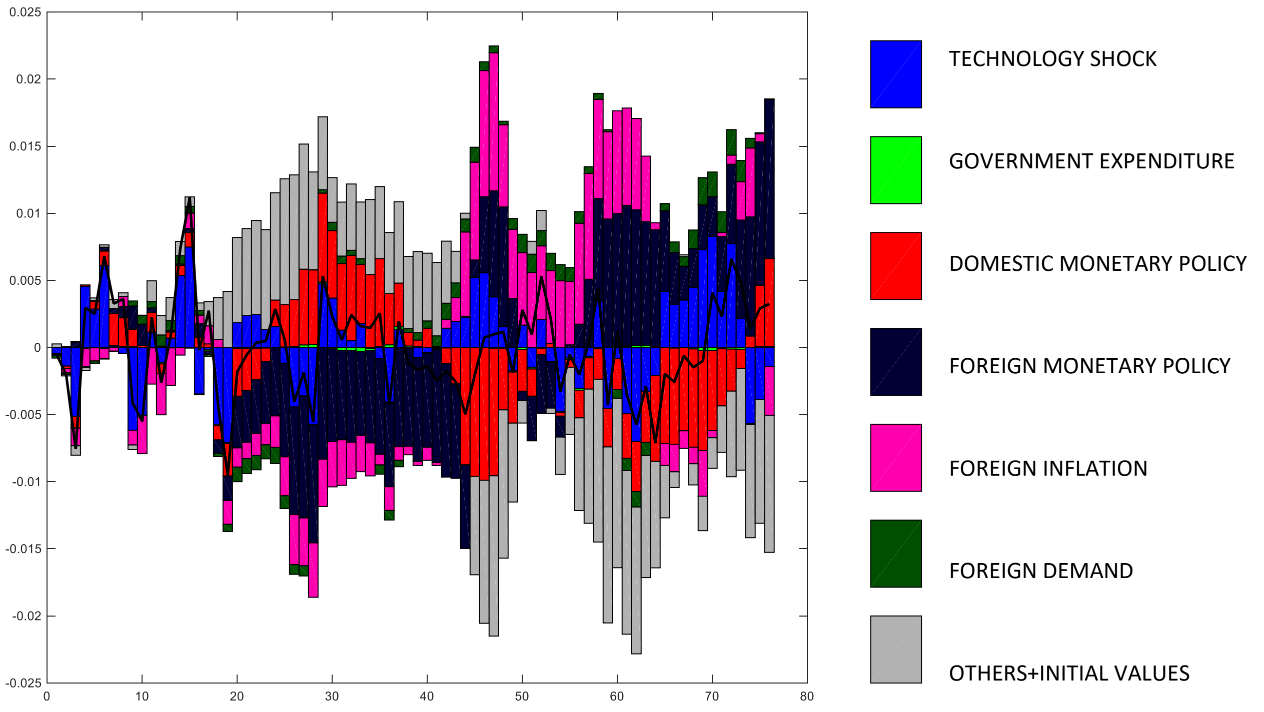
In fact, the output is mainly driven by technology shock followed by external shocks as foreign demand and foreign inflation.

Figure 5: Decomposition variance of output



The historical decomposition of inflation rate shown in Figure (6), offers insights about importance of particular shocks during the specific periods. The model attributes the majority forecast error variance of inflation rate to domestic monetary policy followed by technology shocks.

Figure 6: Decomposition variance of Inflation



6. Conclusion:

In this paper, we estimated and simulated a small open economy DSGE model with financial frictions for the Tunisian Economy. Using Bayesian method, we estimate a key parameter in the accelerator mechanism- elasticity external finance premium with respect to the firm leverage. Results show that external finance premium is statistically significant and is estimated to 0.04, which is close to values used in typical calibrations of DSGE models with a financial accelerator.

Comparing simulated impulse responses function models with financial accelerator to those without a financial accelerator, shows us that the presence of financial frictions amplifies the impact of monetary policy on spreads, investment and the price of capital.

N Interestingly, the model attributes the majority output forecast error variance to external and supply shocks. For inflation, monetary policy shocks are expected to play a relevant role when analysing the sources of fluctuations on the optimal monetary policy.

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